Understanding Parton Distributions from Lattice QCD: Present Limitations and Future Promise

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Workshop on the QCD Structure of the Nucleon

Ferrara April 5, 2002

Outline

Introduction and motivation

Overview of Lattice QCD

Aspects of deep inelastic scattering

Calculation of moments of structure functions on the lattice

Lessons from present lattice calculations

Chiral extrapolation: physics of the pion cloud

Future promise

Summary

Collaborators *LHPC and SESAM collaborations*

MIT

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P. Dreher

A. Pochinsky

BU, JLab, and Florida State

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Adelaide

W. Detmold

W. Melnitchouk

A. Thomas

Introduction

Focus of workshop: QCD structure of the nucleon

Two complementary aspects

- Measuring quark/gluon structure experimentally

  Perturbative QCD

  Precise high energy probe

- Calculating and understanding quark/gluon structure theoretically

  Nonperturbative QCD

  Only method at present is lattice field theory

Essential to do both

Motivation

- Understand structure of nucleon from QCD

- Profound differences between hadrons and
  other many-body systems

  Atoms, molecules, solids, nuclei, ...

  - Constituents can be removed
  - Exchanged boson generating interaction may be
    subsumed into static potential

    photons → Coulomb potential
    mesons → N-N potential

  - Most of mass from fermion constituents

  Nucleons

  - Quarks are confined
  - Gluons are essential degrees of freedom

    Carry half of momentum
    Nonperturbative topological excitations

  - Most of mass generated by interactions

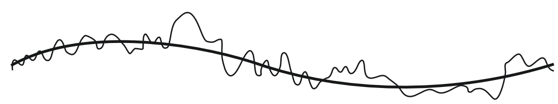
Goals

- Use lattice field theory to **solve** QCD with controlled errors
  - Quantitative calculation of properties of nucleon

    Mass
    Form factors
    Light cone distribution of quark and spin densities

  - Understand "spin crisis"

    Only small fraction of proton spin arises
    from spin of quarks

- Use lattice field theory for **insight** into how QCD works
  - Identify paths that dominate action 
  - Calculate overlap with trial wave function

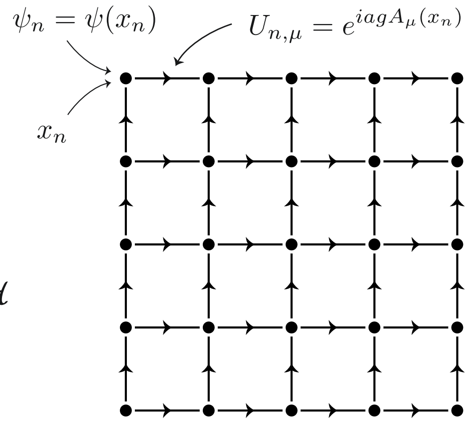
    $$\left|\langle\psi_{\rm trial} \mid \psi_{\rm exact}\rangle\right|^2$$

  - Explore dependence on

    $m_q\ , \quad N_f\ , \quad N_c$

- Determine what is required for definitive comparison with experiment
  - What are the dominant errors with current resources
  - What is required to overcome them

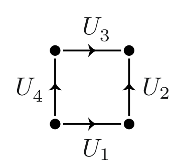
Lattice QCD



Euclidean:

$$e^{i\int dt\, d^3x\, \mathcal{L}} \to e^{-\int d\tau\, d^3x\, \mathcal{H}}$$

$$\begin{aligned}
&\left\langle T\, e^{-\beta H} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi \right\rangle \\
&= \frac{1}{Z}\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\mathcal{D}[A] e^{-\int d^4x \left[\bar\psi(\partial\!\!\!/ + m + igA\!\!\!/)\psi + \frac{1}{4}F_{\mu\nu}^2\right]} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi \\
&\to \prod_n \frac{1}{Z}\int d\psi_n\, d\bar\psi_n\, dU_n e^{-\sum_n \left[\bar\psi M(U)\psi + S(U)\right]} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi \\
&= \prod_n \int dU_n \underbrace{\frac{1}{Z}\det M(U) e^{-S(U)}}_{\text{Sample with M.C.}} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U) \\
&\to \frac{1}{N} \sum_{\substack{i=1 \\ U_i \in \frac{\det M(U)}{Z} e^{-S(U)}}}^{N} M^{-1}(U_i) M^{-1}(U_i) M^{-1}(U_i)
\end{aligned}$$



$$S(U) = \sum_{\square} \frac{2N}{g^2}\left(1 - N^{-1}\mathrm{Re}\mathrm{Tr} U_\square\right) \to \tfrac{1}{4}F_{\mu\nu}^2 \qquad U_\square \equiv U_1 U_2 U_3^\dagger U_4^\dagger$$

$$\bar\psi M(U)\psi = \sum_n \left[\bar\psi_n \psi_n + \kappa(\bar\psi_n(1-\gamma_\mu)U_{n,\mu}\psi_{n+\mu} + \bar\psi_{n+\mu}(1+\gamma_\mu)U_{n,\mu}^\dagger \psi_n\right]$$

Observables

$$\langle T\, e^{-\beta H} \hat{\bar\psi}\hat{\bar\psi}\hat{\bar\psi}\cdots\hat\psi\hat\psi\hat\psi\rangle$$
$$= Z^{-1}\int \mathcal{D}(U)\mathcal{D}(\bar\psi\psi) e^{-\bar\psi M(U)\psi - S(U)}\bar\psi\bar\psi\bar\psi\cdots\psi\psi\psi$$
$$= Z^{-1}\int \mathcal{D}(U) e^{\overbrace{\text{ln det}\, M(U)}^{②} - \overbrace{S(U)}^{③}} \sum \underbrace{M^{-1}(U)M^{-1}(U)\cdots M^{-1}(U)}_{①}$$

① $M^{-1} = (1+\kappa U)^{-1}$ connects $\bar\psi$ and ψ with line of U's

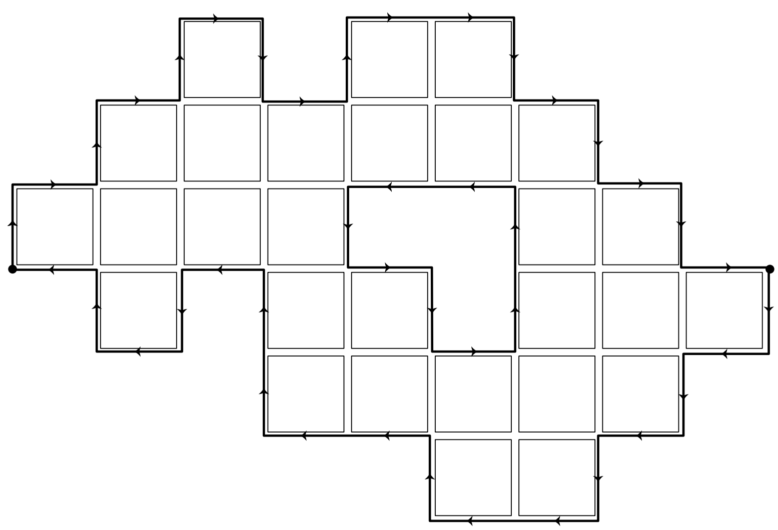
→ Sum over all valence quark paths.

② ln det M generates closed loops of U's

→ Sum over all $\bar q q$ excitations from sea
omit in quenched approximation

③ $S(U)$ tiles with plaquettes

→ sum all gluons



$32^3 \times 64$ lattice $\Longrightarrow 10^8$ gluon variables

Essential Insights from Lattice QCD

Two distinct regimes with different physics:

- Heavy quark regime

  Confinement

  Flux tubes

  Adiabatic potential

  Isgur Wise function

- Light quark regime

  Chiral symmetry breaking

  Instantons

  Zero modes: $\langle \bar{\psi}\psi \rangle = \pi \rho(0)$

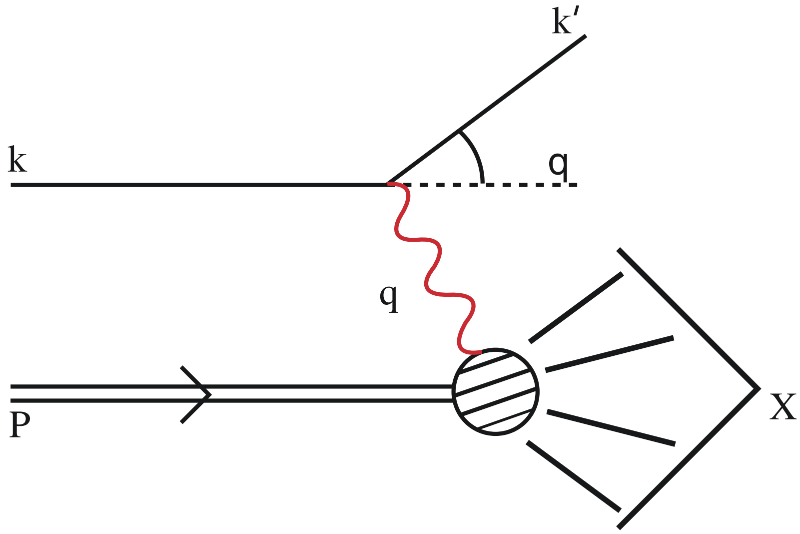
  Quarks propagate via 't Hooft interaction

  Zero modes dominate quark propagation

  Instantons alone yield observables similar to those from all gluons

  Low energy effective theory - chiral perturbation theory

Deep Inelastic Scattering



$$e(k) + N(P) \to e(k') + X$$

$$Q^2 \stackrel{\text{def}}{=} -q^2 = 4EE' \sin^2 \frac{\theta}{2} > 0$$
$$\nu \stackrel{\text{def}}{=} P \cdot q = M(E - E')$$
$$x \stackrel{\text{def}}{=} Q^2/2\nu$$

- Cross section determined by hadronic tensor $W_{\mu\nu}$

$$\left|\frac{A}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$
$$W_{\mu\nu} = \frac{1}{4\pi} \sum_X \langle PS|J^\mu|X\rangle (2\pi)^4 \delta^{(4)}(P + q - P_X) \langle X|J^\nu|PS\rangle$$
$$= \int d^4\xi \, e^{iq\xi} \, \langle PS|[J^\mu(\xi), J^\nu(0)]|PS\rangle$$

- Unpolarized scattering measures symmetric part with 2 structure functions:

$$W_{\{\mu\nu\}} = (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) F_1(\nu, Q^2) + \frac{1}{\nu}[(P^\mu - \frac{\nu}{q^2} q^\mu)(P^\nu - \frac{\nu}{q^2} q^\nu)] F_2(\nu, Q^2)$$

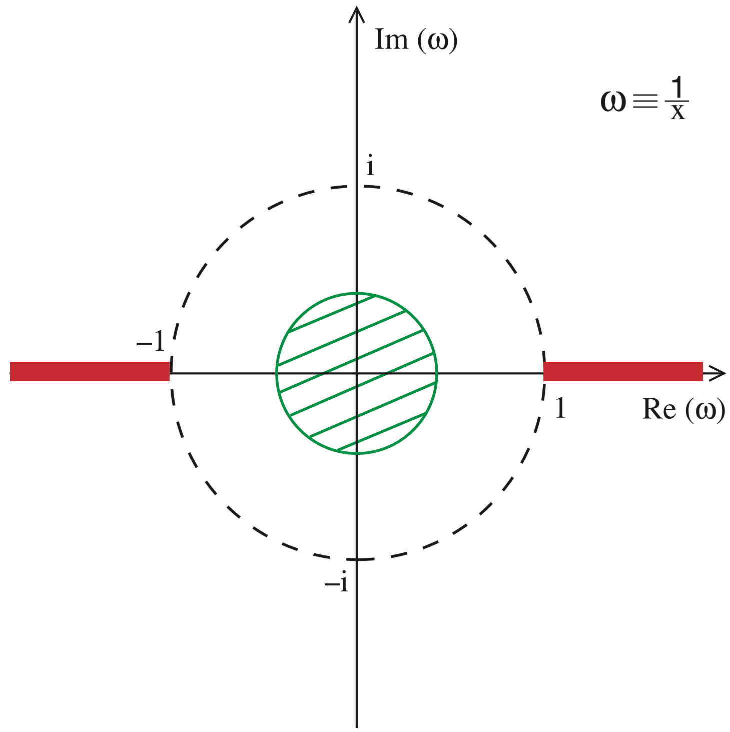
- Polarized scattering measures antisymmetric part with 2 structure functions:

$$W_{[\mu\nu]} = -i\epsilon^{\mu\nu\lambda\rho} q_\lambda (\frac{S_\rho}{\nu}(g_1(\nu, Q^2) + g_2(\nu, Q^2)) - \frac{q \cdot S P_\rho}{\nu^2} g_2(\nu, Q^2))$$

- In parton model

$$F_1 = \tfrac{1}{2} \sum_q e_q^2 (q^\uparrow(x) + q^\downarrow(x)) \qquad F_2 = 2xF_1 \qquad g_1 = \tfrac{1}{2} \sum_q e_q^2 (q^\uparrow(x) - q^\downarrow(x)) \qquad g_2 = 0$$

Sketch of Operator Product Expansion



Forward Compton amplitude

$$T_{\mu\nu}(\nu, q^2) = i \int d^4\xi \; e^{iq\xi} \; \langle P|T(J^\mu(\xi)J^\nu(0))|P\rangle$$

Dispersion relation, $\omega = 1/x$

$$W(\omega, q^2) = \frac{1}{4\pi} \, \mathrm{Im}\, T(\omega, q^2)$$

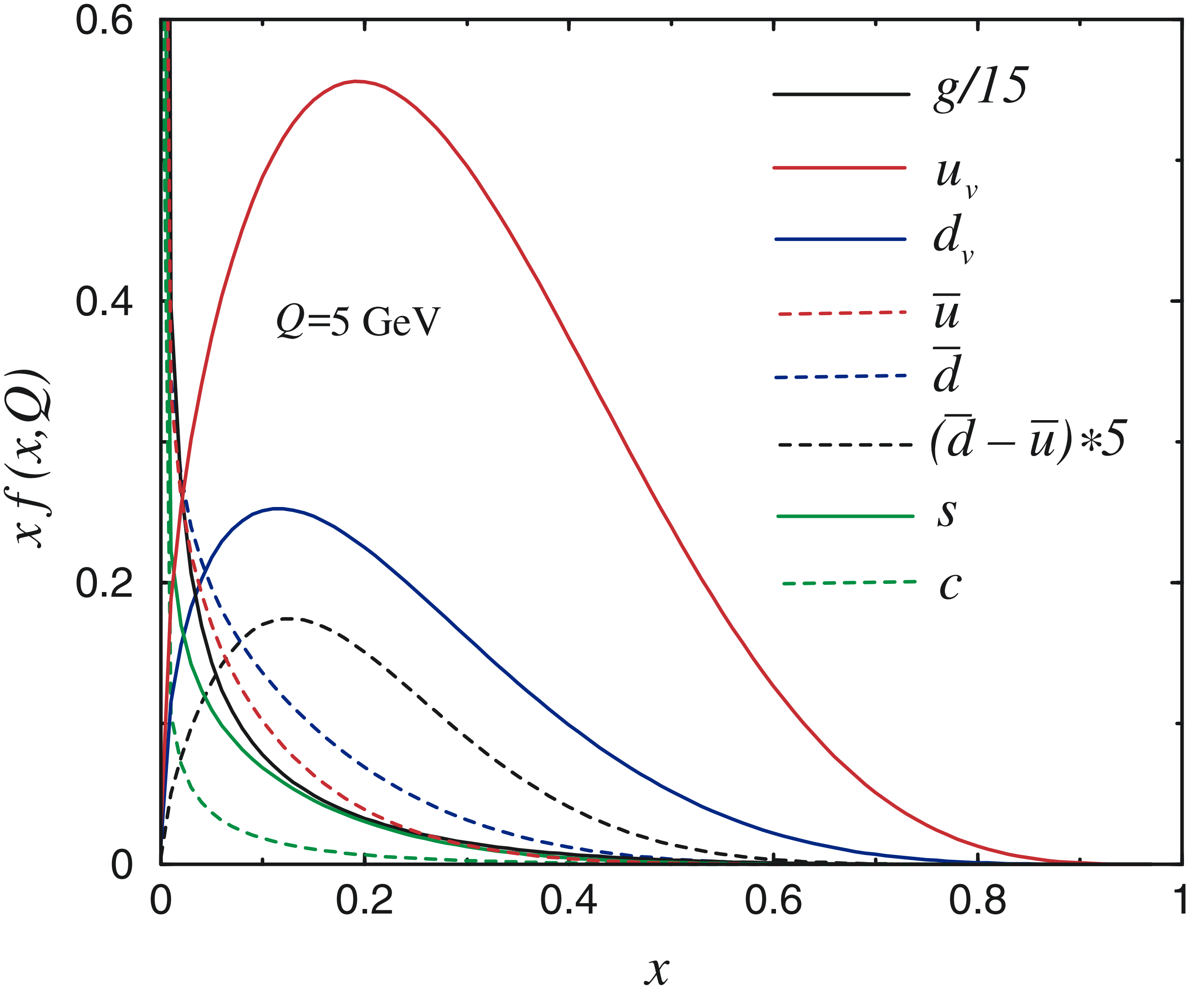
$$T(\omega, q^2) = 4 \int_1^\infty d\omega' \, \omega' \, \frac{W(\omega', q^2)}{\omega'^2 - \omega^2}$$

$$\sum_n C_n \frac{q_{\mu_1} \cdots q_{\mu_n}}{Q^{2n}} \langle P|\bar\psi\gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_n\}} \psi|P\rangle = 4 \sum_{\text{even } n} \omega^n \int_1^\infty d\omega' \, (\omega')^{n-1} W(\omega', q^2)$$

$$\sum_n C_n \Big(\frac{2p\cdot q}{Q^2}\Big)^n \qquad \langle x^{n-1}\rangle_q \qquad = 4 \sum_{\text{even } n} x^{-n} \int_0^1 dx \, x^{n-1} W(x, q^2)$$

$$C_n \langle x^{n-1}\rangle_q \propto \int_0^1 dx \, x^{n-1} W(x, q^2)$$

Experimentally measured quark and gluon distributions at $Q^2 = 5$ GeV



Phenomenological fits to unpolarized data

CTEQ: http://www-spires.dur.ac.uk/hepdata/cteq.html

GRV: http://www-spires.dur.ac.uk/hepdata/grv.html

MRS: http://durpdg.dur.ac.uk/hepdata/mrs.html

Phenomenological fits to polarized data

GRSV: http://doom.physik.uni-dortmund.de/PARTON/index.html

GS:http://www.desy.de/ gehrt/pdf

Moments of quark and gluon distributions

Moments of quark distributions in the proton

$$\langle x^n \rangle_q = \int_0^1 \mathrm{d}x\, x^n \big(q(x) + (-1)^{n+1}\bar{q}(x)\big)$$
$$\langle x^n \rangle_{\Delta q} = \int_0^1 \mathrm{d}x\, x^n \big(\Delta q(x) + (-1)^{n}\Delta\bar{q}(x)\big)$$
$$\langle x^n \rangle_{\delta q} = \int_0^1 \mathrm{d}x\, x^n \big(\delta q(x) + (-1)^{n+1}\delta\bar{q}(x)\big)$$

where $\quad q = q_\uparrow + q_\downarrow \qquad \Delta q = q_\uparrow - q_\downarrow \qquad \delta q = q_\top + q_\perp$

are related to matrix elements of twist-2 operators

$$\frac{1}{2}\big\langle PS\big|\bar{\psi}\gamma^{\{\mu_1} iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = 2\langle x^{n-1}\rangle_q\, P^{\{\mu_1}\cdots P^{\mu_n\}}$$

$$\big\langle PS\big|\bar{\psi}\gamma^{\{\mu_1}\gamma_5 iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{2}{n}\langle x^{n-1}\rangle_{\Delta q}\, S^{\{\mu_1} P^{\mu_2}\cdots P^{\mu_n\}}$$

$$\big\langle PS\big|\bar{\psi}\sigma^{[\alpha\{\mu_1]}\gamma_5 iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{2}{M}\langle x^{n-1}\rangle_{\delta q}\, S^{[\alpha} P^{\{\mu_1]} P^{\mu_2}\cdots P^{\mu_n\}}$$

where $\{\,\} \Rightarrow$ symmetrization, $[\,] \Rightarrow$ antisymmetrization, and $S^2 = M^2$

Higher twist operators:

$$\big\langle PS\big|\bar{\psi}\gamma^{[\mu_1}\gamma_5 iD^{\{\mu_2]}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{1}{n} d_{n-1}\, S^{[\mu_1} P^{\{\mu_2]}\cdots P^{\mu_n\}}$$

Lattice Operators

Use irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

$\langle x\rangle_q^{(a)}$	6_3^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$	p
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar\psi\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar\psi\gamma_i\overleftrightarrow{D}_i\psi$	0
$\langle x^2\rangle_q$	8_1^-	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$	p, m
$\langle x^3\rangle_q$	2_1^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar\psi\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3 \leftrightarrow 4\}$	p, m^*
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar\psi\gamma^5\gamma_3\psi$	0
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$	p
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$	0
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$	p
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar\psi\gamma^5\sigma_{34}\psi$	0
$\langle x\rangle_{\delta q}$	8_1^-	$\bar\psi\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$	p
d_1	6_1^+	$\bar\psi\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$	$0, M$
d_2	8_1^-	$\bar\psi\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$	p, M

where $(\bar\psi\overleftrightarrow{D}\psi)_n = \bar\psi_n U_{n,\mu}\psi_{n+\mu} - \bar\psi_{n-\mu}U^\dagger_{n-\mu,\mu}\psi_n$

m ⇒ mixing with same dimension operators

m^* ⇒ no mixing for Wilson or overlap

M ⇒ mixing with lower dimension for Wilson, not for overlap

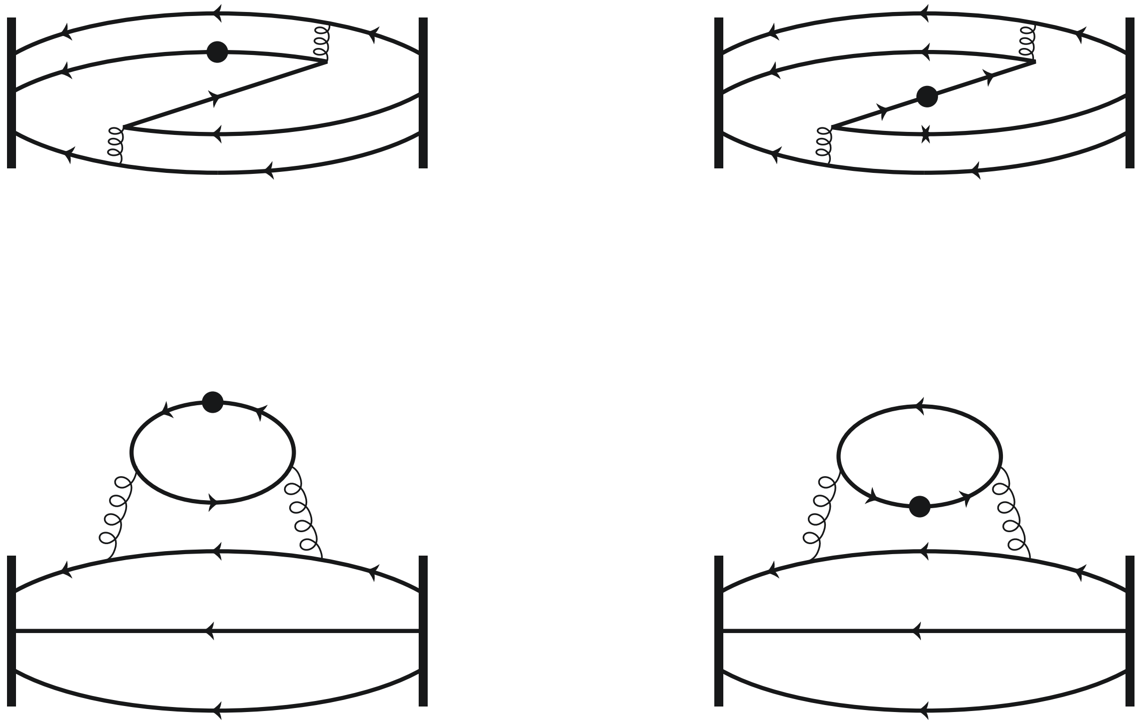
Perturbative Renormalization

$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left(\gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

observable	γ	B^{LATT}	$B^{\overline{MS}}$	$Z(\beta = 6.0)$	$Z(\beta = 5.6)$
$\langle x \rangle_q^{(a)}$	8/3	-3.16486	-40/9	0.9892	0.9884
$\langle x \rangle_q^{(b)}$	8/3	-1.88259	-40/9	0.9784	0.9768
$\langle x^2 \rangle_q$	25/6	-19.57184	-67/9	1.1024	1.1097
$\langle x^3 \rangle_q$	157/30	-35.35192	-2216/225	1.2153	1.2307
$\langle 1 \rangle_{\Delta q}$	0	15.79628	0	0.8666	0.8571
$\langle x \rangle_{\Delta q}^{(a)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x \rangle_{\Delta q}^{(b)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x^2 \rangle_{\Delta q}$	25/6	-19.56159	-67/9	1.1023	1.1096
$\langle 1 \rangle_{\delta q}$	1	16.01808	-1	0.8563	0.8461
$\langle x \rangle_{\delta q}$	3	-4.47754	-5	0.9956	0.9953
d_1	0	0.36500	0	0.9969	0.9967
d_2	7/6	-15.67745	-35/18	1.1159	1.1242

Note mixing of $d_n \propto \gamma_5 \gamma_{[\sigma} \overleftrightarrow{D}_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ with $\frac{1}{a} \gamma_5 \gamma_{[\sigma} \gamma_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ for Wilson fermions.

Hadron Matrix Elements on Lattice



- Measure $\langle \mathcal{O} \rangle$, for m_q, a, L
- Connected diagrams

  $p = 0$

  $p \neq 0$
- Disconnected diagrams
- Extrapolate

  $m_q : m_\pi \to 140\,\text{MeV}$

  $a \to \sim 0.05\,\text{fm}$

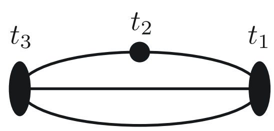
  $L \to \sim 5.0\,\text{fm}$
- Note: For $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$, disconnected diagrams cancel

Calculation of Matrix Elements on Euclidean Lattice

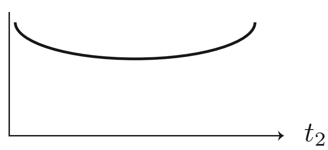
$J^\dagger$: Current with quantum numbers of proton

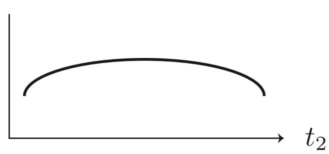
$\big|\psi_J\big\rangle = J^\dagger\big|\Omega\big\rangle$ Trial function

$$\big\langle T J(t_3)\, \mathcal{O}(t_2)\, J^\dagger(t_1)\big\rangle = \sum_{m,n}\big\langle\psi_J\big|n\big\rangle\big\langle n\big|\mathcal{O}\big|m\big\rangle\big\langle m\big|\psi_J\big\rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



$$\xrightarrow[\substack{t_3-t_2\gg 1\\ t_2-t_1\gg 1}]{} \big|\big\langle\psi_J\big|0\big\rangle\big|^2\big\langle 0\big|\mathcal{O}\big|0\big\rangle e^{-E_0(t_3-t_1)}$$





want $\big|\big\langle\psi_J\big|n\big\rangle\big|^2 \sim \delta_{n0}$

for best plateau

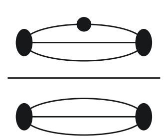
Normalize:

$$\big\langle T J(t_3)\, J^\dagger(t_1)\big\rangle = \sum_n\big|\big\langle\psi_J\big|n\big\rangle\big|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1\gg 1]{} \big|\big\langle\psi_J\big|0\big\rangle\big|^2 e^{-E_0(t_3-t_1)}$$

$\Longrightarrow$

$$\big\langle 0\big|\mathcal{O}\big|0\big\rangle = \frac{\langle J\mathcal{O}J^\dagger\rangle}{\langle JJ^\dagger\rangle} =$$

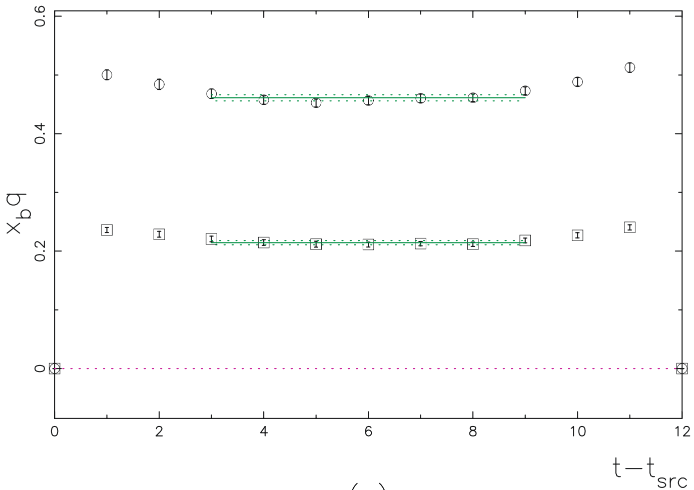


Sequential source options:

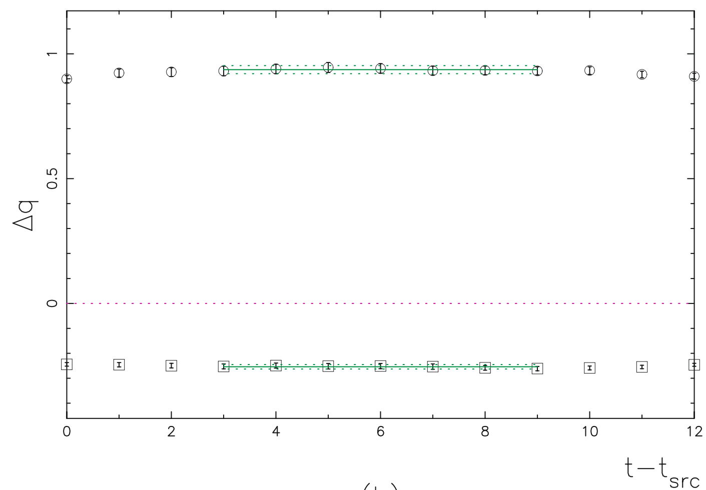
Each $\mathcal{O}(2)S(2,1)$ generates $S(2,3)$ to all t_3

Sink at fixed t_3 generates $S(3,2)$

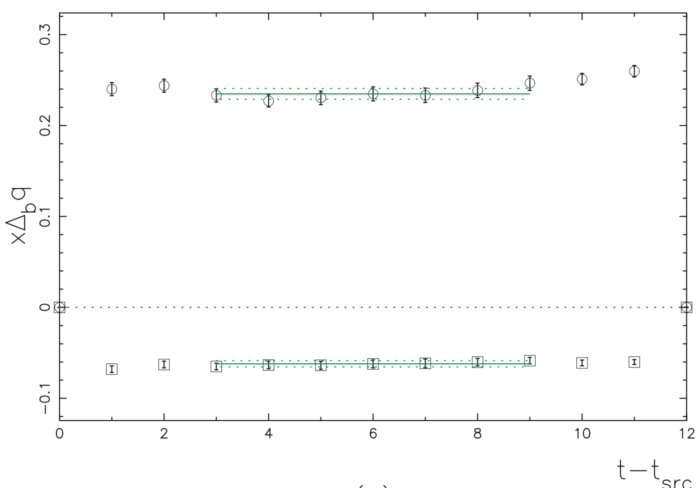
Plateaus in full QCD for operators with $\mathbf{p = 0}$



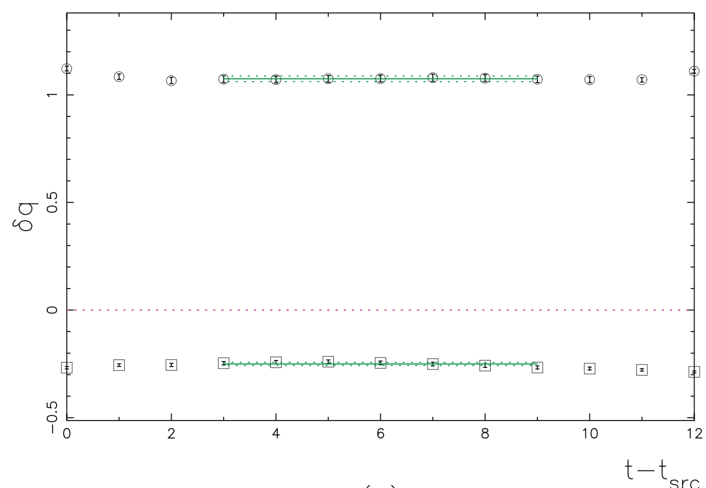
(a)



(b)



(c)



(d)

Lessons from Present Lattice Calculations

Lattice implementation consistent

Qualitative behavior given by instantons

Quenched vs full QCD

- Agree for $m_\pi \geq 500$ MeV
- Wrong chiral logs for small m_π

Extrapolation to continuum limit: a small

- Improved action yields small $\mathcal{O}(a)$ errors
- Extrapolation of Wilson and improved action agree

Extrapolation to large volume: L large

- Traditional $L = 1.6$ fm too small - 15 % effect on g_A

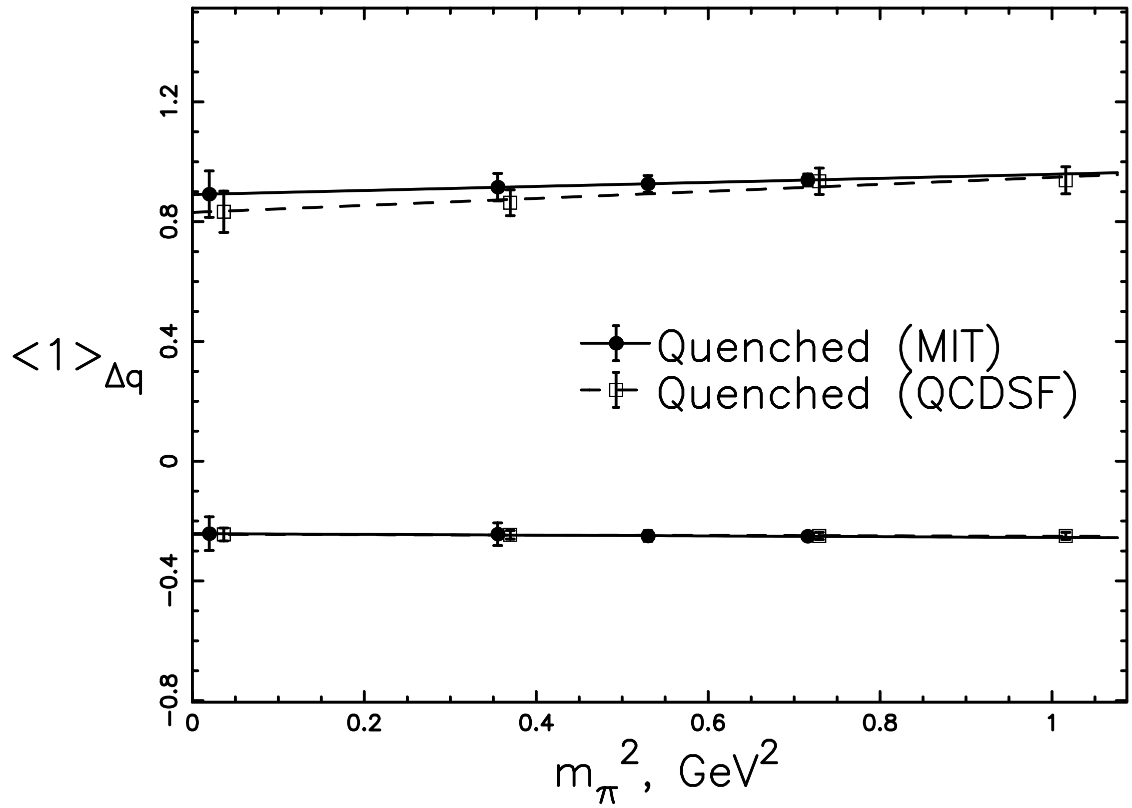
Extrapolation to chiral limit: m_π small

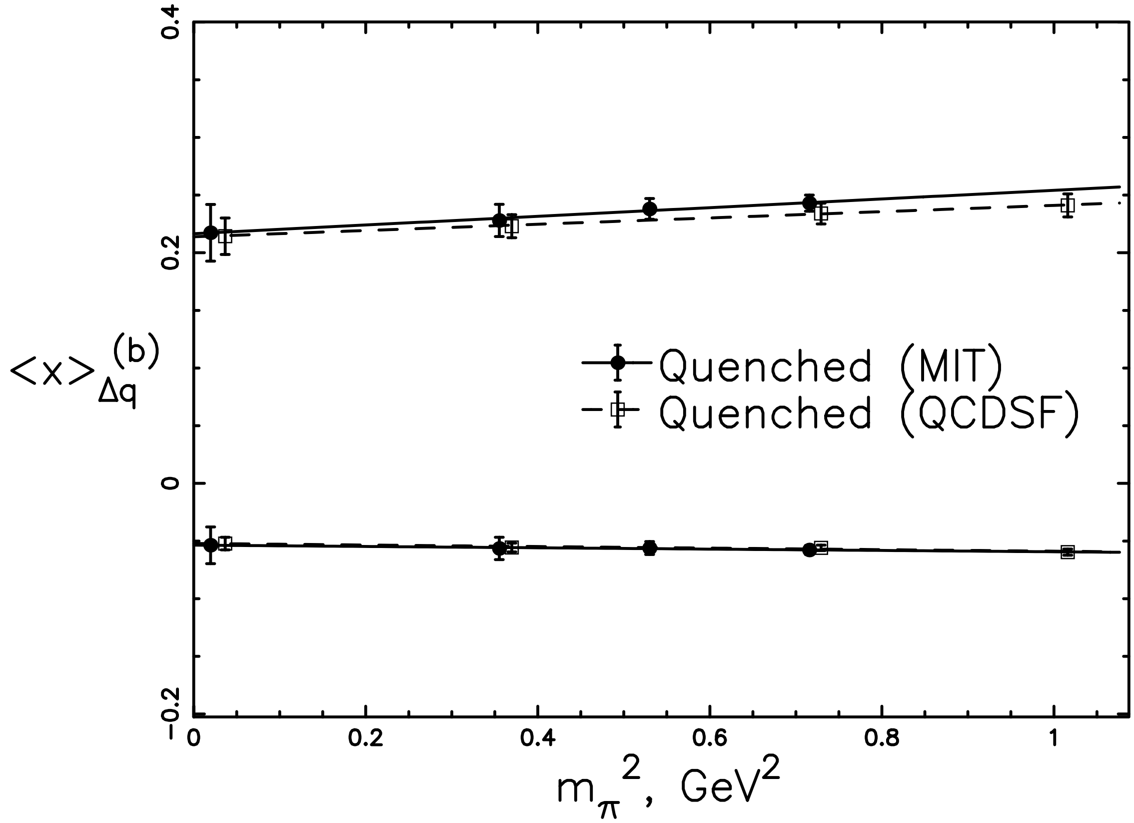
- **Primary issue for future**

Naive chiral extrapolation inconsistent with phenomenology

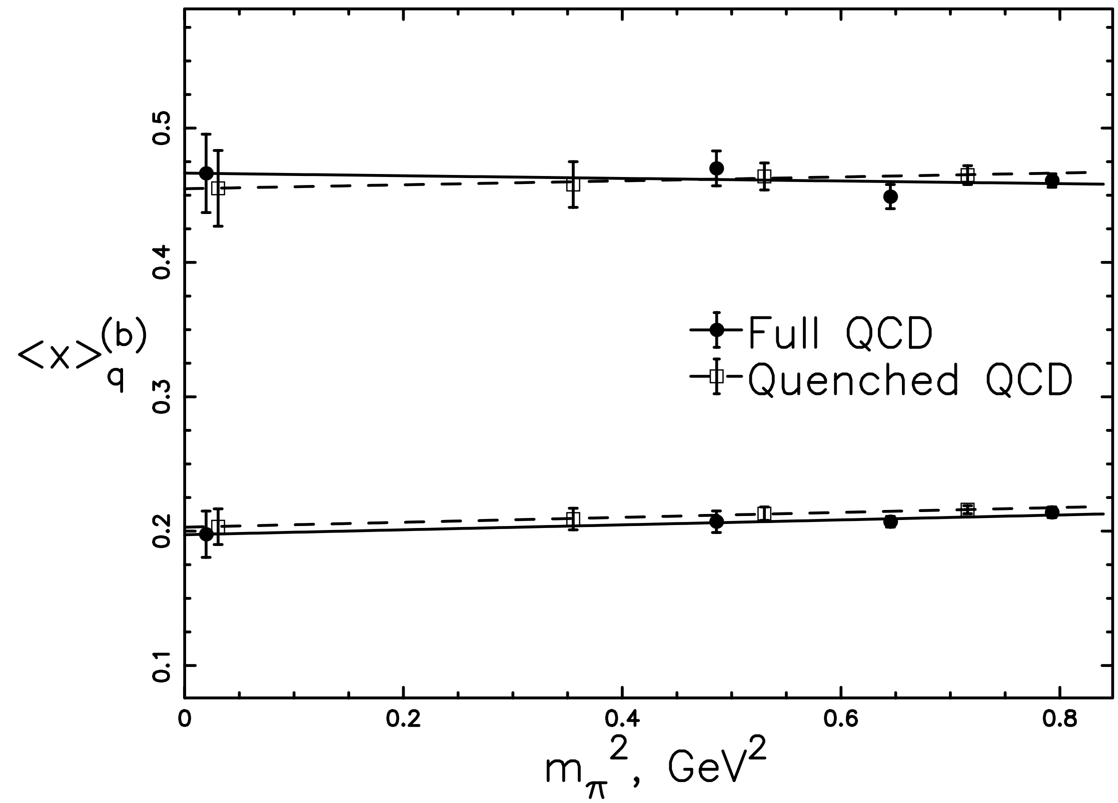
- Discrepancy of order 50% for $\langle x^n \rangle_q$

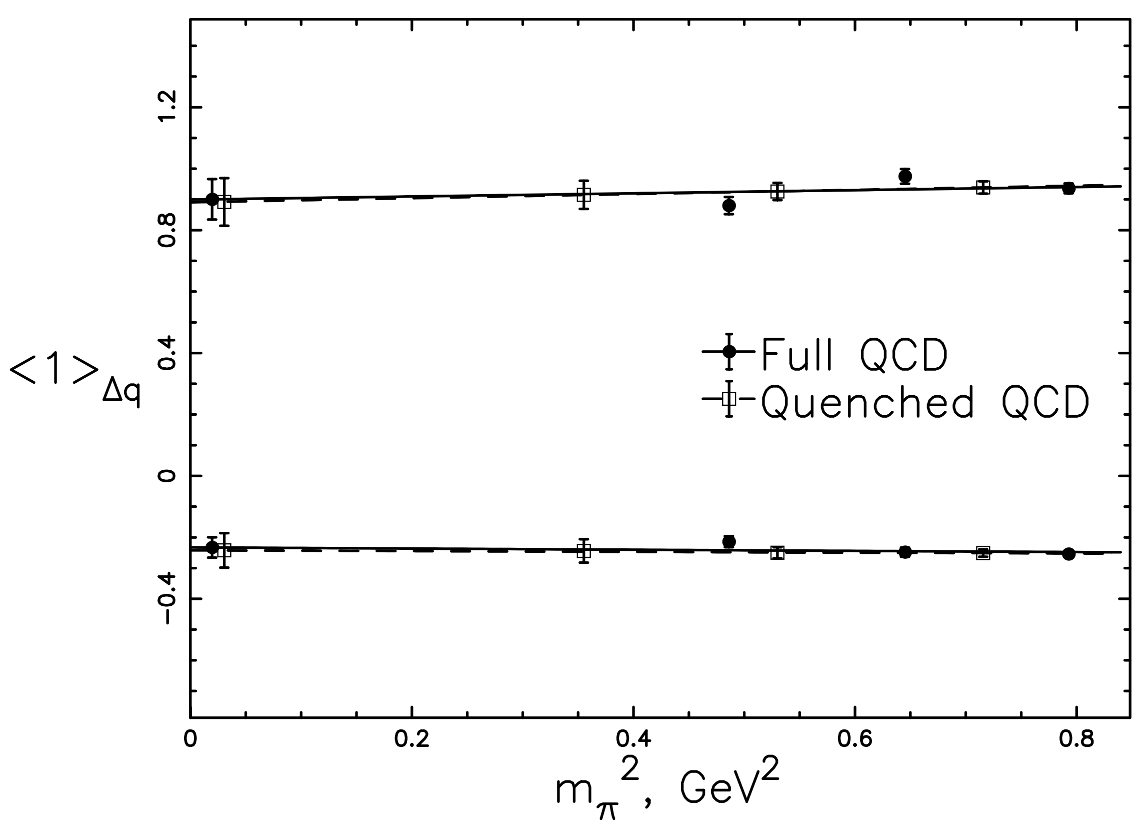
Comparison of Quenched Calculations



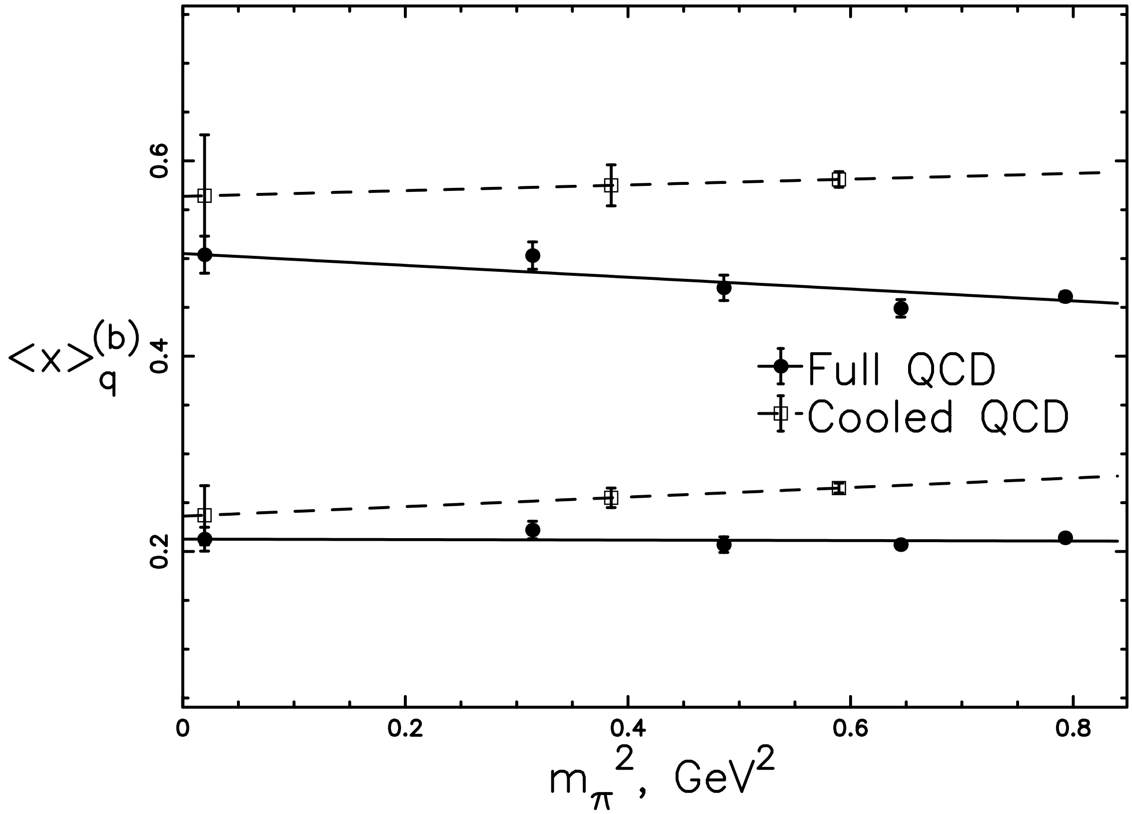


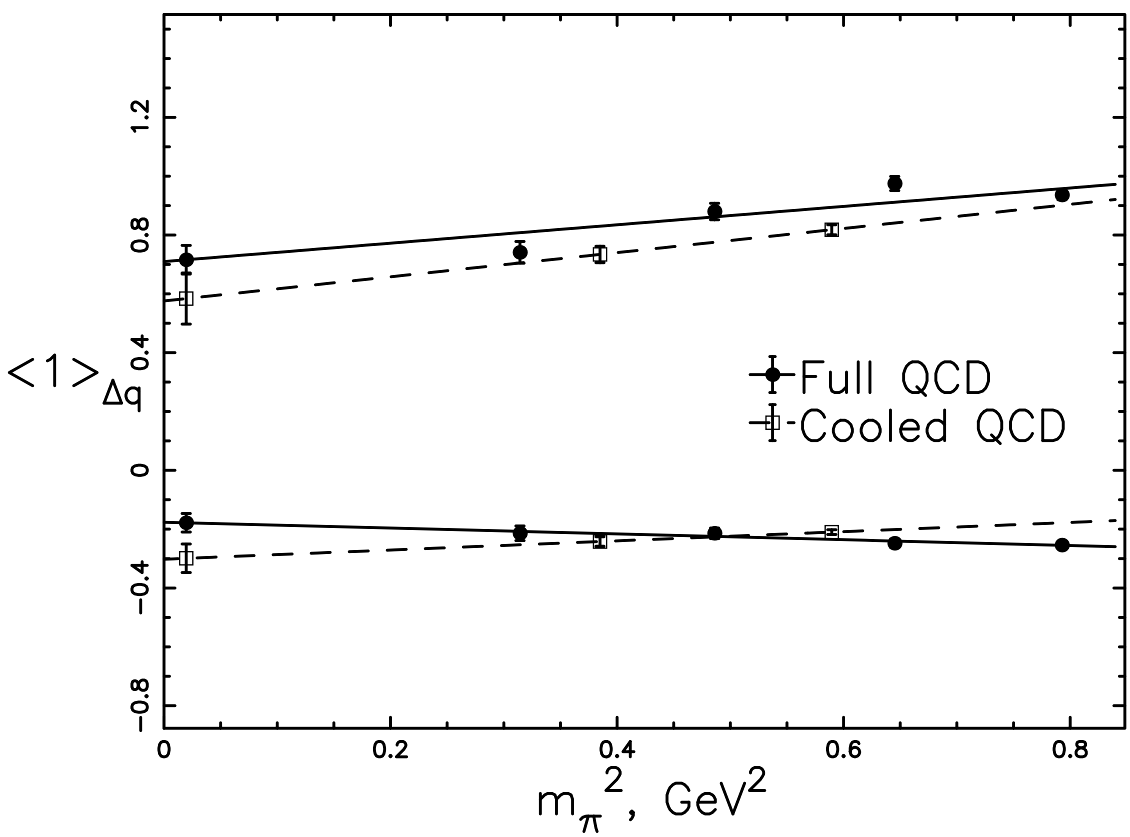
Comparison of Full and Quenched QCD





Qualitative Behavior from Instantons





Comparison with Other Calculations and Phenomenology

Connected M. E.	QCDSF	QCDSF ($a=0$)	Wuppertal	Quenched	Full QCD (3 pts)	Phenomenology ($q \pm \bar{q}$)
$\langle x \rangle_u$	0.452(26)			0.454(29)	0.459(29)	
$\langle x \rangle_d$	0.189(12)			0.203(14)	0.190(17)	
$\langle x \rangle_{u-d}$	0.263(17)			0.251(18)	0.269(23)	0.154(3)
$\langle x^2 \rangle_u$	0.104(20)			0.119(61)	0.176(63)	
$\langle x^2 \rangle_d$	0.037(10)			0.029(32)	0.031(30)	
$\langle x^2 \rangle_{u-d}$	0.067(22)			0.090(68)	0.145(69)	0.055(1)
$\langle x^3 \rangle_u$	0.022(11)			0.037(36)	0.069(39)	
$\langle x^3 \rangle_d$	−0.001(7)			0.009(18)	−0.010(15)	
$\langle x^3 \rangle_{u-d}$	0.023(13)			0.028(49)	0.078(41)	0.023(1)
$\langle 1 \rangle_{\Delta u}$	0.830(70)	0.889(29)	0.816(20)	0.888(80)	0.860(69)	
$\langle 1 \rangle_{\Delta d}$	−0.244(22)	−0.236(27)	−0.237(9)	−0.241(58)	−0.171(43)	
$\langle 1 \rangle_{\Delta u-\Delta d}$	1.074(90)	1.14(3)	1.053(27)	1.129(98)	1.031(81)	1.248(2)
$\langle x \rangle_{\Delta u}$	0.198(8)			0.215(25)	0.242(22)	
$\langle x \rangle_{\Delta d}$	−0.048(3)			−0.054(16)	−0.029(13)	
$\langle x \rangle_{\Delta u-\Delta d}$	0.246(9)			0.269(29)	0.271(25)	0.196(9)
$\langle x^2 \rangle_{\Delta u}$	0.087(14)			0.027(60)	0.116(42)	
$\langle x^2 \rangle_{\Delta d}$	−0.025(6)			−0.003(25)	0.001(25)	
$\langle x^2 \rangle_{\Delta u-\Delta d}$	0.112(15)			0.030(65)	0.115(49)	0.061(6)
δu_c	0.93(3)	0.980(30)		1.01(8)	0.963(59)	
δd_c	−0.20(2)	−0.234(17)		−0.20(5)	−0.202(36)	
d_2^u	−0.206(18)			−0.233(86)	−0.228(81)	
d_2^d	−0.035(6)			0.040(31)	0.077(31)	

Chiral Extrapolation - Physics of the Pion Cloud

- Long-standing puzzle: Linear extrapolation in m_q
  yields serious discrepancies

  $\langle x \rangle_u - \langle x \rangle_d \ \sim \ 0.24 - 0.28 \quad (0.16)$

  $g_A = \langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d} \sim 1.0 - 1.1 \qquad (1.26)$

- Pion cloud essential component of nucleon

  μ_N, g_A

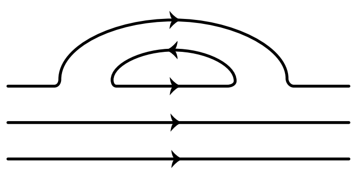
  Suppressed with heavy quarks in small volume

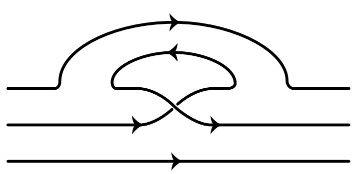
  Require:

  Light quarks

  Large volume: $L \geq 4 \frac{1}{m_\pi}$

  Full QCD





Chiral Perturbation Theory

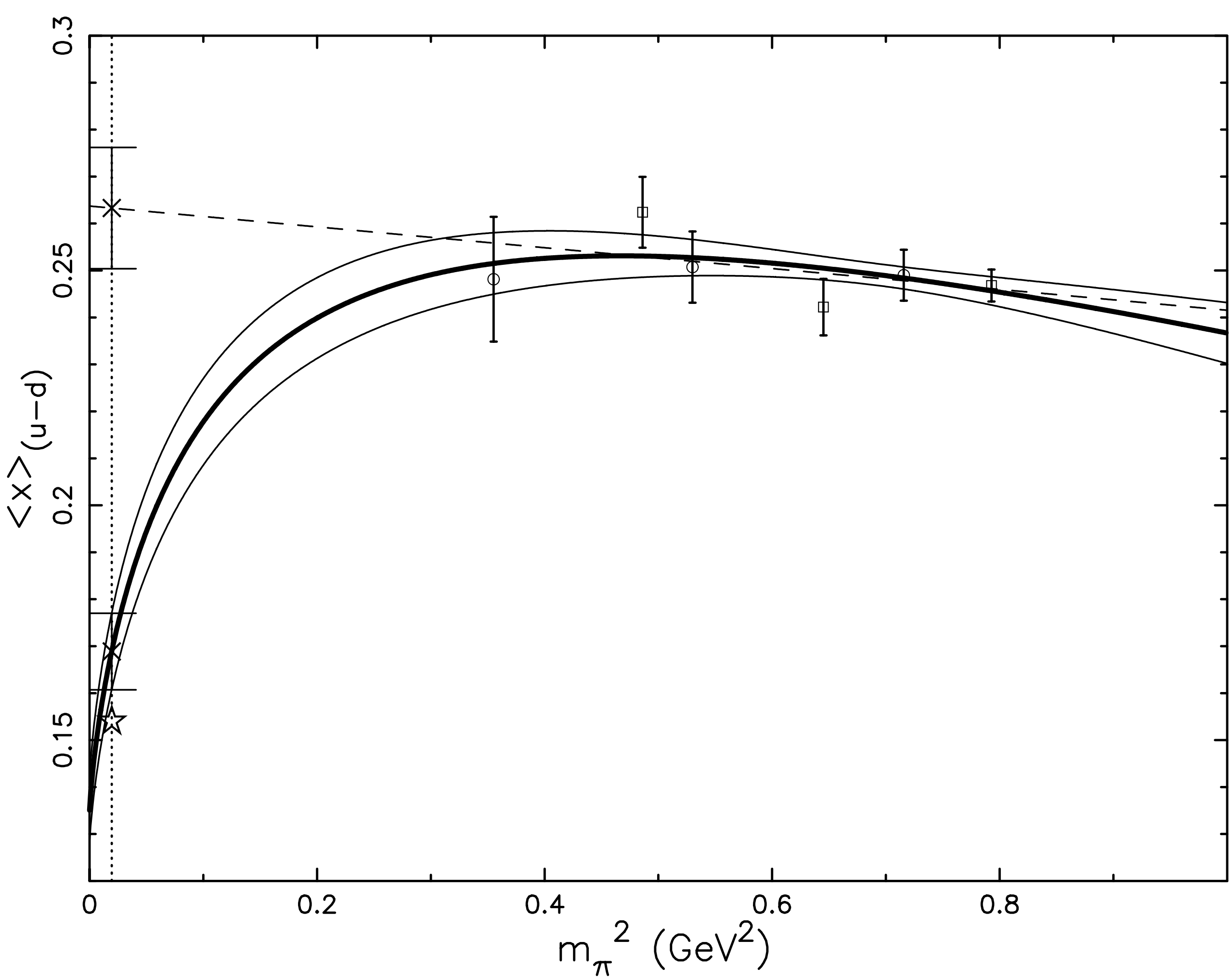
Heavy baryon chiral perturbation theory for nucleon parton distributions

Chen & Ji, Arndt & Savage, Chen & Savage

$$\langle x^n\rangle_u - \langle x^n\rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln(m_\pi^2)\right] + \text{analytic terms}$$

- Physical chiral extrapolation formula hep-lat/0103006

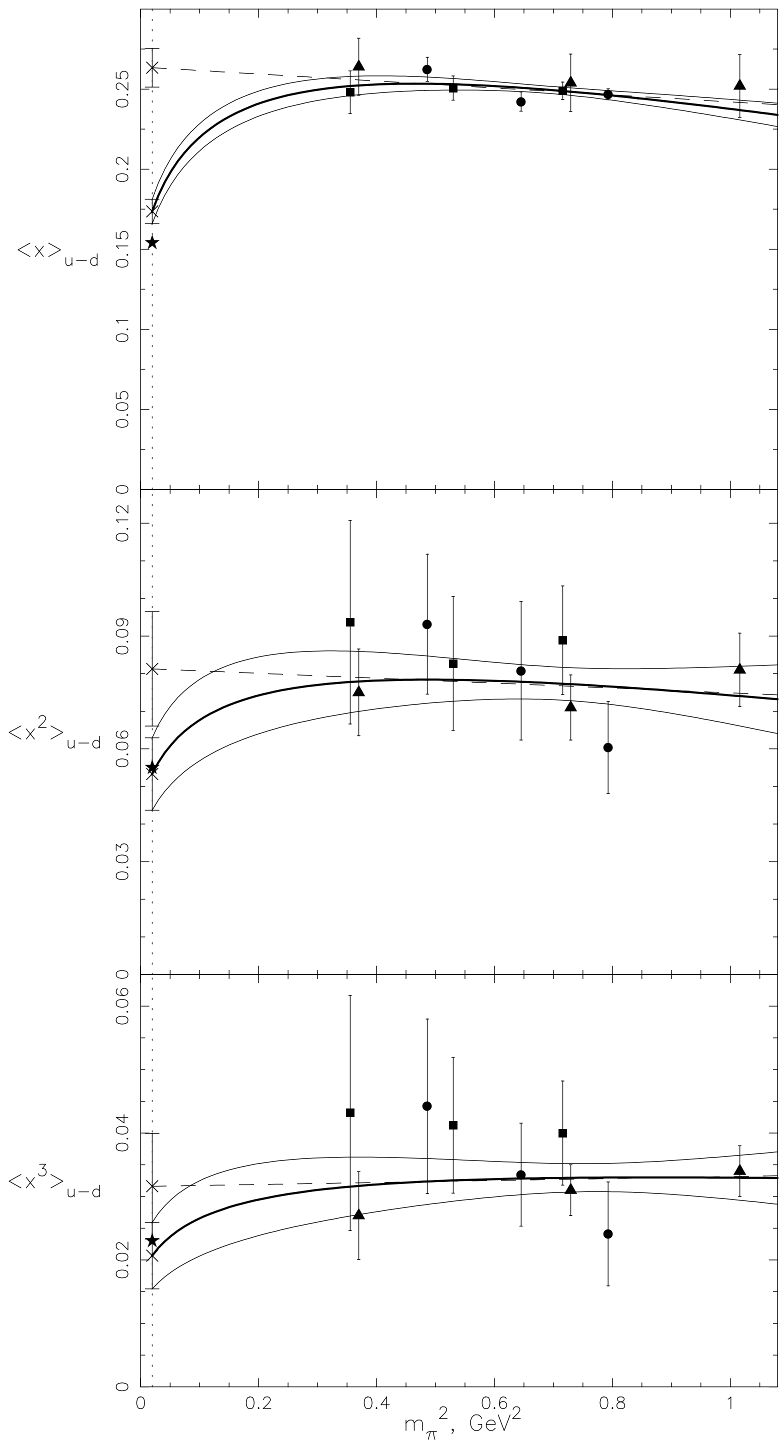
$$\langle x^n\rangle_u - \langle x^n\rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln\Big(\frac{m_\pi^2}{m_\pi^2+\mu^2}\Big)\right] + b_n m_\pi^2$$



Squares full QCD, circles quenched, $\mu = 550 MeV$

Consistent results for $\mu \sim 550$ MeV

$$\langle x \rangle_u - \langle x \rangle_d \ , \quad \langle x^2 \rangle_u - \langle x^2 \rangle_d \ , \quad \langle x^3 \rangle_u - \langle x^3 \rangle_d$$



Diamonds full QCD, squares MIT quenched, triangles QCDSF quenched

Chiral Perturbation Theory *(cont.)*

Spin distribution

$$\langle x^n \rangle_{\Delta u} - \langle x^n \rangle_{\Delta d} \sim a_n \left[1 - \frac{(2{g_A}^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln(m_\pi^2) \right] + \text{analytic terms}$$

Significant contributions from Δ excitation

Large N_C estimate: cancel $\sim$ 60 % of chiral log

Quenched QCD

Ghost loops cancel sea quark loops

Alters coefficient of chiral logs

Introduces spurious double pole from η'

Partially Quenched QCD

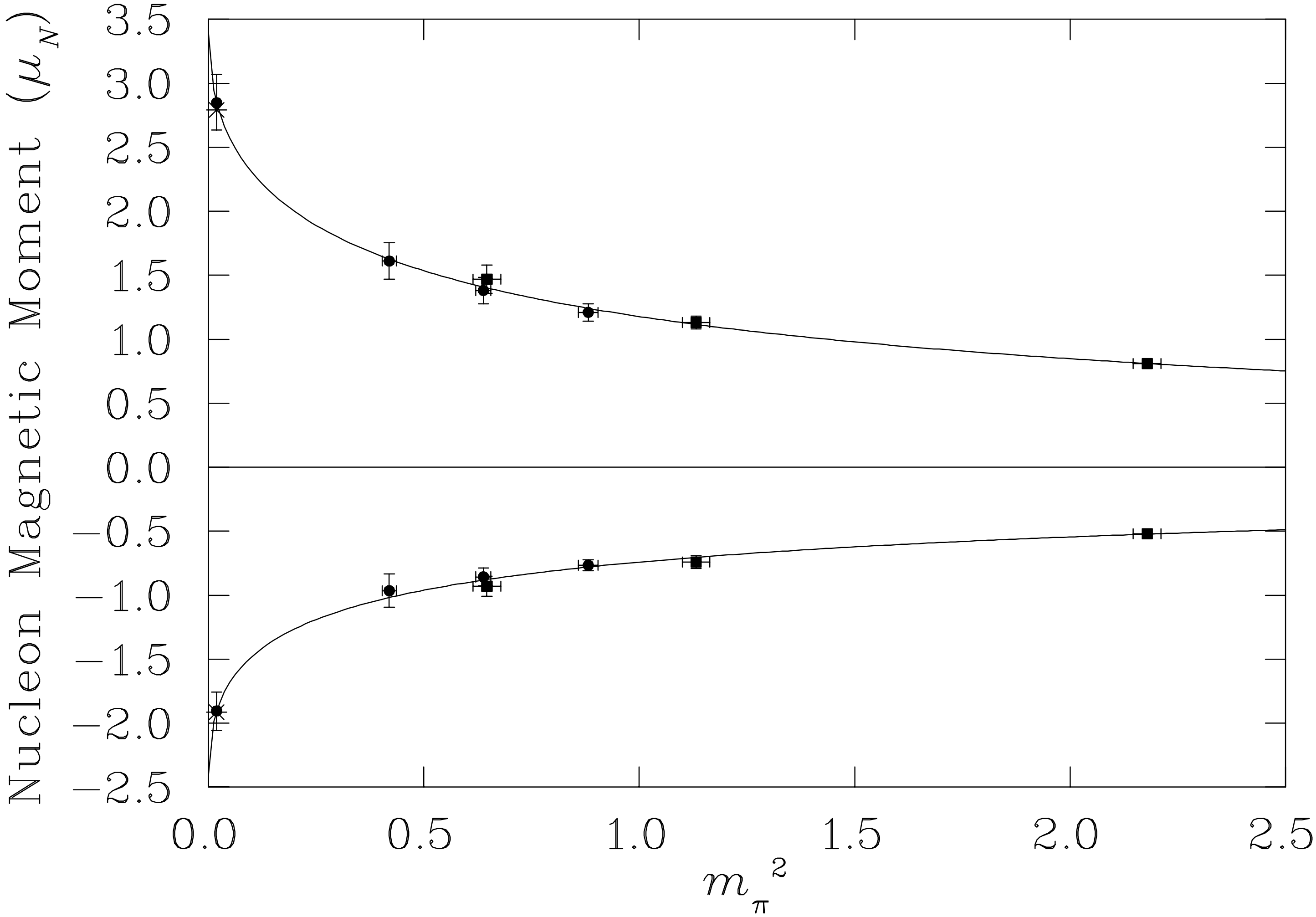
Chiral logs for general m_{sea} and $m_{valence}$

Tool to extrapolate to $m_{sea} = m_{valence} = m_{physical}$

Incorporate finite volume, finite Q corrections

Analogous result seen for magnetic moment

D. Leinweber, D. Lu, and A. Thomas hep-lat/0103006



Future Promise

- Have theoretical framework for definitive calculations
- Improved actions - continuum limit
- Chiral fermions - exact lattice chiral symmetry
- Systematic partially quenched chiral expansion

  Measure parameters of effective theory

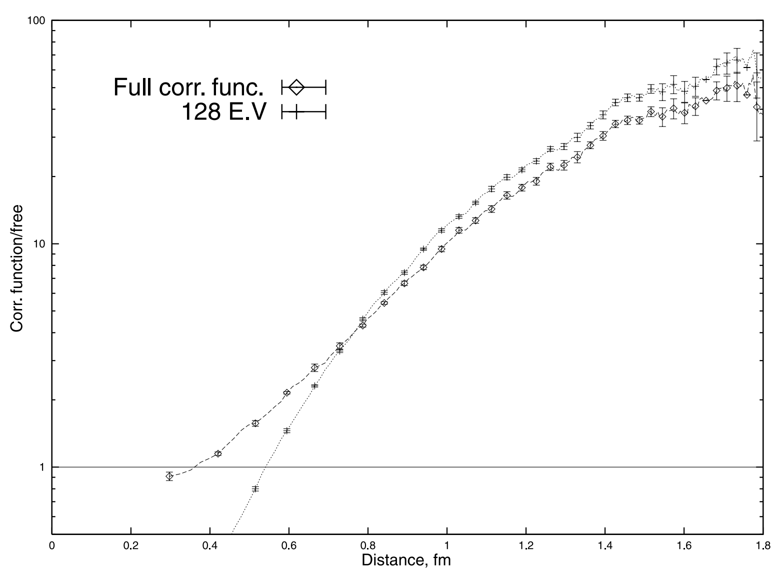
  Hybrid combination of chiral valence quarks, staggered sea quarks

  Corrections for finite volume, fixed topology
- Calculate in chiral regime

  Decrease m_q

  Increase L
- Nonperturbative renormalization
- Gluon distributions
- Calculate disconnected diagrams

  Exploit zero mode dominance - eigenmode expansion



Definitive Calculation Requires Terascale Calculation

5% measurement at m_π^2 =0.05 GeV^2

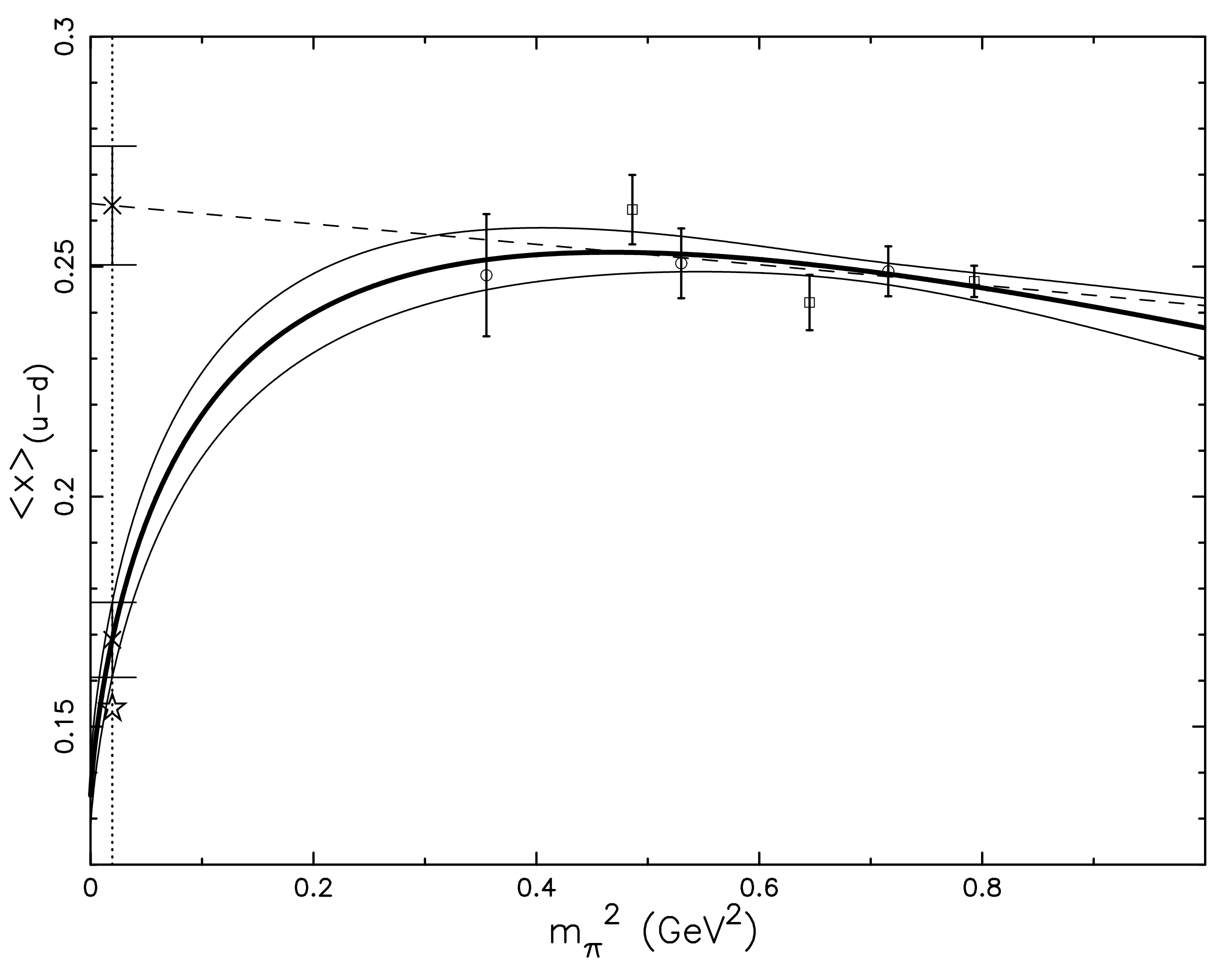
$$m_\pi \sim 230 MeV$$

$$L \sim 4.3 fm.$$

SESAM cost function

$$N_{\mathrm{OPS}} \sim 0.38 \Big[\frac{L}{4}\Big]^{4.55} \Big[\frac{0.8}{a}\Big]^{7.25} \Big[\frac{0.3}{m_\pi/m_\rho}\Big]^{2.7}$$

~ 8 Tflops-years



Summary

- Key issue is chiral extrapolation - physics of pion cloud
- Definitive calculations of moments of parton distributions are possible with 10's of sustained Teraflops
- Understanding hadrons structure requires both:

  Facilities for experimental measurements and

  Facilities for fundamental lattice calculations
- Should think of dedicated computers in the same way as one thinks of experimental apparatus for the field
- Should think of international cooperation in large-scale computation in the same way as in large-scale experiments