Theoretical Accuracy of Vub from Exclusive Decays

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* Covering for B. Grinstein

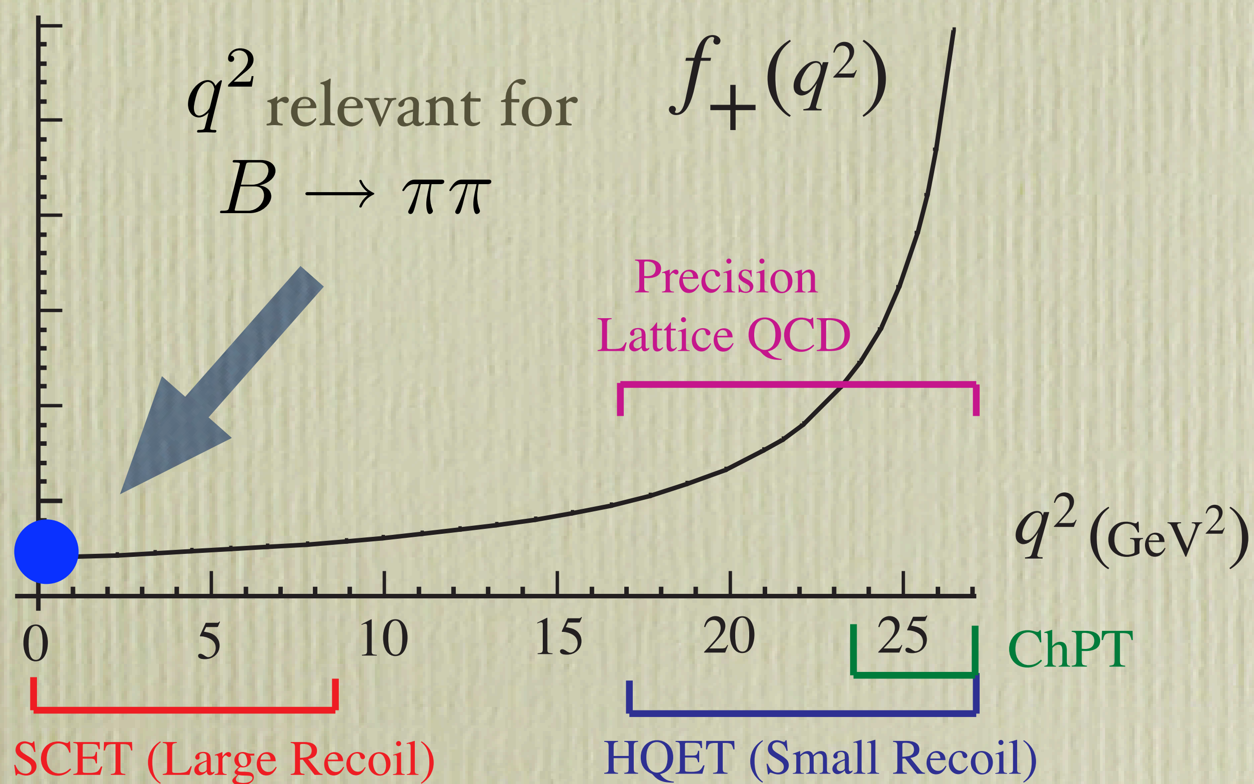
Goal: A precision model independent determination of Vub from exclusive decays

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 \, |f_+(q^2)|^2$$

(neglecting lepton mass)

$$\langle \pi(p_\pi)|\bar{u}\gamma^\mu b|B(p_B)\rangle = \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu\right) f_+(q^2) + q^\mu \frac{m_B^2 - m_\pi^2}{q^2} f_0(q^2)$$

J.Dingfelder (CKM'05)



Average from Cleo, Belle, Babar :

$\mathrm{Br}(B \to \pi \ell \bar{\nu})$ uncertainty is 9%

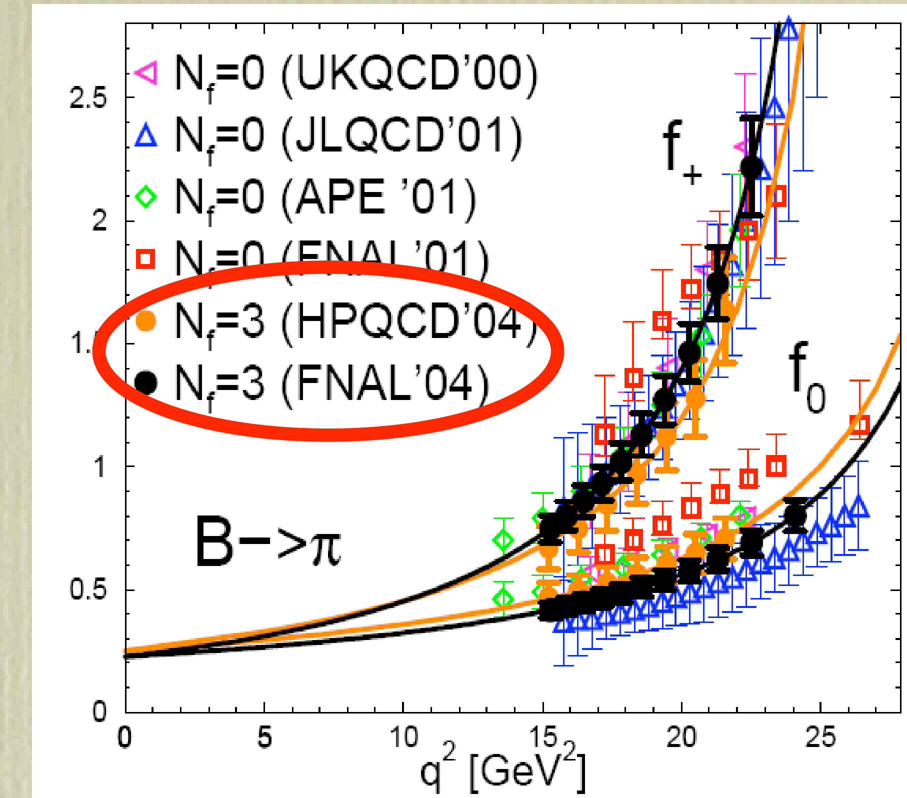
→ $|V_{ub}|$ to 4.5% !

Uncertainty from theory dominates ...

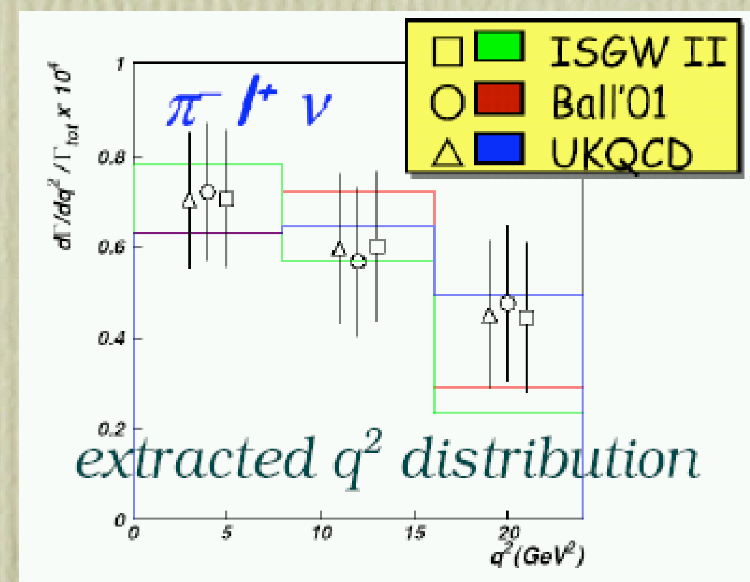
Unquenched Lattice QCD

f_+ has: 11% systematic error

$\sim$ 7% statistical errors



Cleo:



$q^2 \geq 16\,\mathrm{GeV}^2$

20% expt. error

$E_\pi \leq 1\,\mathrm{GeV}$

$$|V_{ub}| = (3.52 \pm 0.44 \pm 0.73) \times 10^{-3} (\mathrm{HPQCD})$$

$$|V_{ub}| = (3.0 \pm 0.4 \pm 0.6) \times 10^{-3} (\mathrm{FNAL})$$

Belle:

$$|V_{ub}|^{\pi\ell\nu}_{(q^2 \geq 16)} = (3.87 \pm 0.70 \pm 0.22^{+0.85}_{-0.51}) \times 10^{-3} \quad (\mathrm{FNAL'04}),$$

$$|V_{ub}|^{\pi\ell\nu}_{(q^2 \geq 16)} = (4.73 \pm 0.85 \pm 0.27^{+0.74}_{-0.50}) \times 10^{-3} \quad (\mathrm{HPQCD}).$$

Can the total error be reduced below 25% using the full range of q^2?

A precision model independent exclusive Vub:

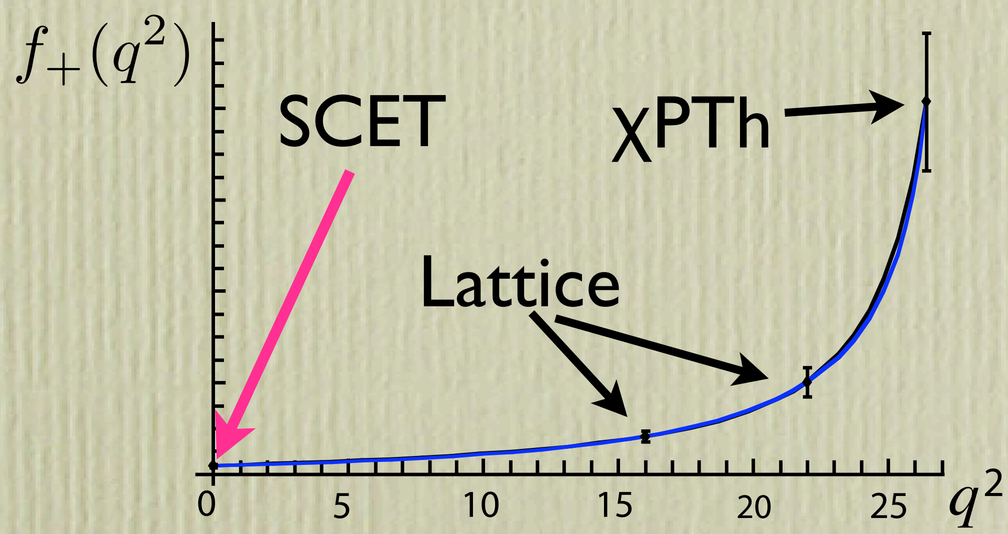
Arnesen, Grinstein, Rothstein, I.S.
(hep-ph/0504209)

Use: i) lattice qcd results at large q^2

ii) chiral perturbation theory at $q^2_{\rm max}$

iii) SCET constraint from $B \to \pi\pi$ at $q^2 = 0$

iv) QCD dispersion relations to constrain the form factor shape between input points



(disp. relations are also used for excl. Vcb)

Method I for Vub: use only total Br

Method II for Vub: incorporate information from q^2 spectra

Dispersion Relations

M.N Meiman, '63
S. Okubo, I. Fushih, '71
V. Singh, A. K. Raina, '79
C. Bourrely, B. Machet, E de Rafael, '81
E. de Rafael, J. Taron, '92 & '94
B. Grinstein, P. Mende, '93
C.G. Boyd, B. Grinstein, R. Lebed, '95, '96, '97
L. Lellouch, '96
C.G. Boyd, M. Savage, '97
I. Caprini, L. Lellouch, M. Neubert '97
M. Fukunaga, T. Onogi, '05

Define

$$\Pi_J^{\mu\nu}(q) = \frac{1}{q^2}(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L(q^2) \equiv i\int d^4x\, e^{iqx}\langle 0|\mathrm{T} J^\mu(x) J^{\dagger\nu}(0)|0\rangle$$

Dispersion relations

$$\chi^{(0)} = \frac{1}{2}\frac{\partial^2 \Pi_J^T}{\partial(q^2)^2}\bigg|_{q^2=0} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^T(t)}{t^3}$$

Inequality

$$\mathrm{Im}\Pi_J^{T,L} = \frac{1}{2}\sum_X (2\pi)^4\delta^4(q-p_X)|\langle 0|J|X\rangle|^2 \geq \pi(2\pi)^3\delta^4(q-p_B-p_\pi)|\langle 0|J|B\pi\rangle|^2$$

Perturbative QCD (OPE)

Related by crossing to decay form factor

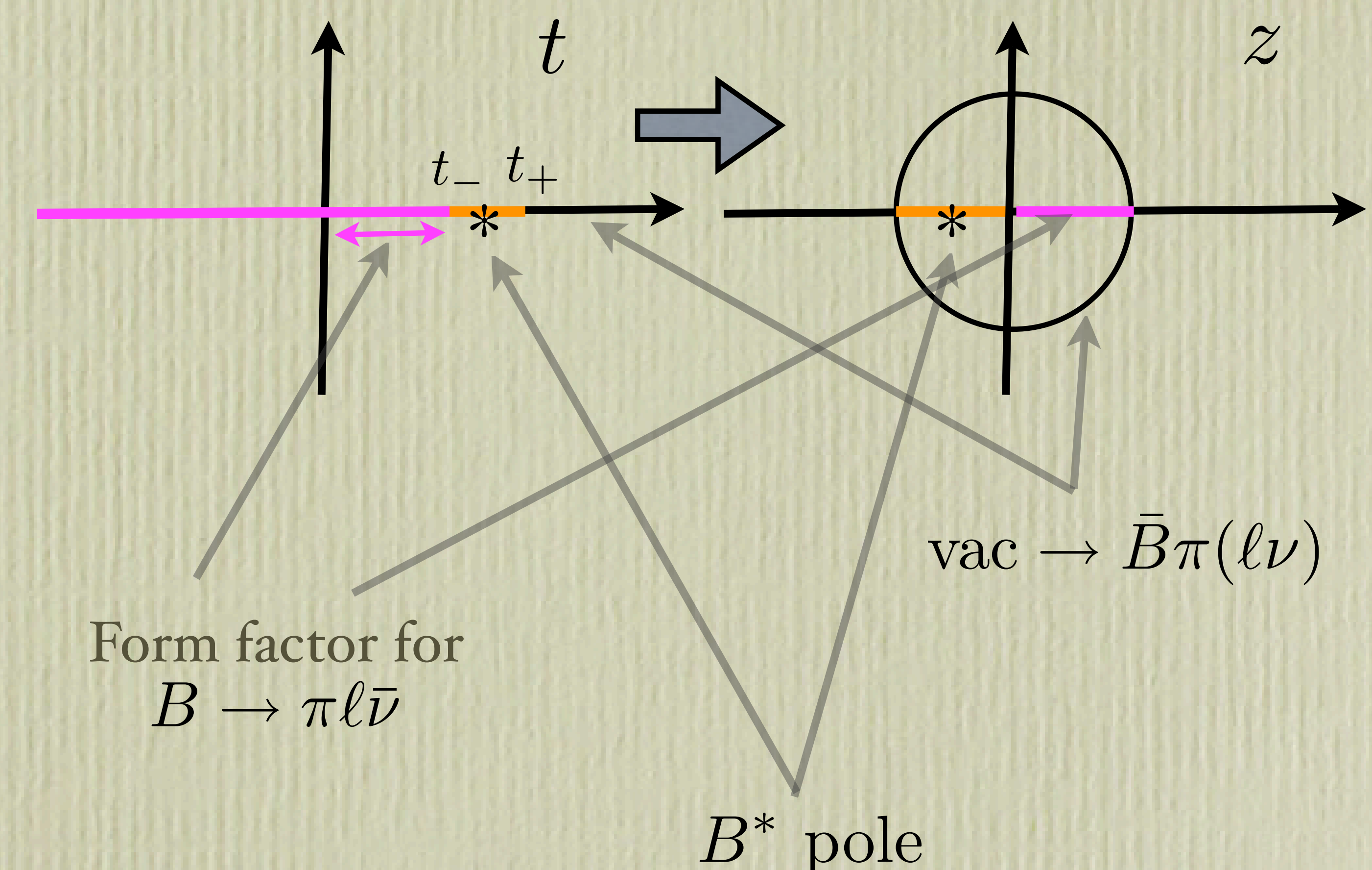
Bound on Form factor

$$\int_{t_+}^\infty dt\, \frac{W(t)|f(t)|^2}{t^3} \leq 1$$

$$t_+ = (m_B + m_\pi)^2$$

Complex Magic

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \qquad t_\pm = (m_B \pm m_\pi)^2$$



$$P(t)\phi(t)f(t) = \sum_{n=0}^{\infty} a_n\, z^n$$

Blaschke Factor: remove pole at $t = m_{B^*}^2$

Outer function: phase space, Jacobian, $\chi^{(0)}$ in QCD

$t = q^2$

$$f_+(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n\, z^n$$

Pick $t_0 = 0.65\, t_-$ then

$$-0.34 \le z \le 0.22$$

Strategy: use input points to fix first few a's
vary higher a's to determine bounds

Input Points

$$|V_{ub}| f_+(0) = \left[\frac{64\pi}{m_B^3 f_\pi^2} \frac{\overline{B}r(B^- \to \pi^0\pi^-)}{\tau_{B^-}|V_{ud}|^2 G_F^2}\right]^{1/2} \times \left[\frac{(C_1+C_2)t_c - C_2}{C_1^2 - C_2^2}\right]\left[1 + \mathcal{O}\Big(\alpha_s(m_b), \frac{\Lambda_{\rm QCD}}{m_b}\Big)\right],$$

$$t_c = t_c(S_{\pi^+\pi^-}, C_{\pi^+\pi^-}, {\rm Br}(\pi^+\pi^-), {\rm Br}(\pi^0\pi^-), \beta, \gamma)$$

Bauer, Pirjol, Rothstein, I.S.
(B.B.N.S; Luo & Rosner)

i) SCET

$$f_{\rm in}^0 = |V_{ub}| f_+(0) = (7.2 \pm 1.8) \times 10^{-4}$$

ii) Lattice (FNAL /MILC)

$$f_{\rm in}^1 = f_+(15.87)$$
$$f_{\rm in}^2 = f_+(18.58)$$
$$f_{\rm in}^3 = f_+(24.09)$$

take systematic error to be 100% correlated

$$E_{ij} = \sigma_i^2 \delta_{ij} + y^2 f_{\rm in}^i f_{\rm in}^j$$

iii) Chiral Pert. Theory

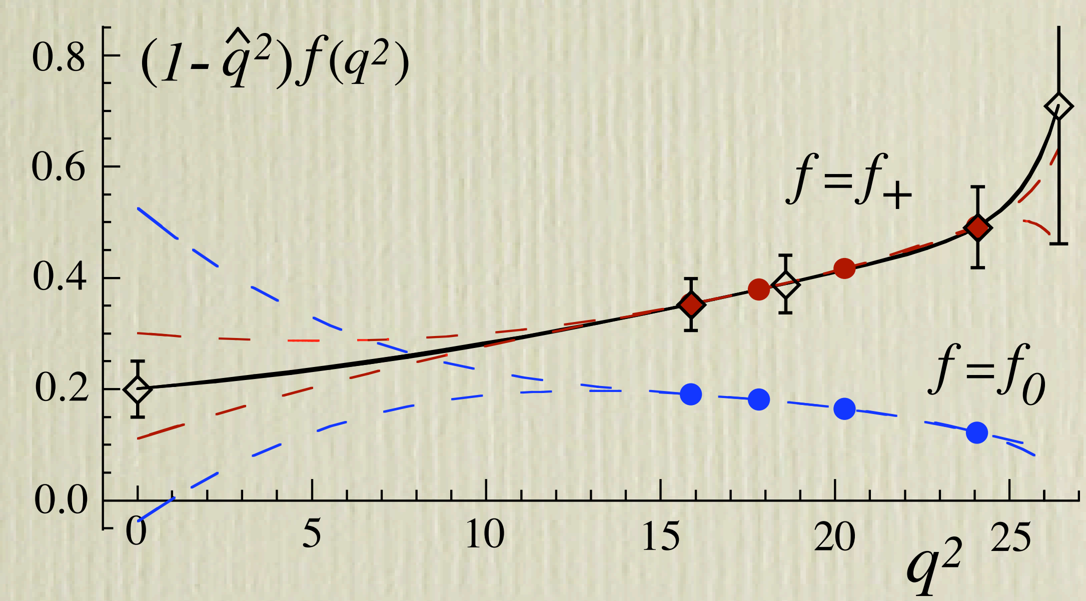
$$f_+(q^2(E_\pi)) = \frac{g f_B m_B}{2 f_\pi (E_\pi + m_{B^*} - m_B)}\left[1 + \mathcal{O}\Big(\frac{E_\pi}{\Delta}\Big)\right] \qquad \Delta \sim 600\,\text{MeV}$$

$$f_{\rm in}^4 = f_+(26.42) = 10.38 \pm 3.63$$

$$\hat{q}^2 = q^2/m_{B^*}^2$$

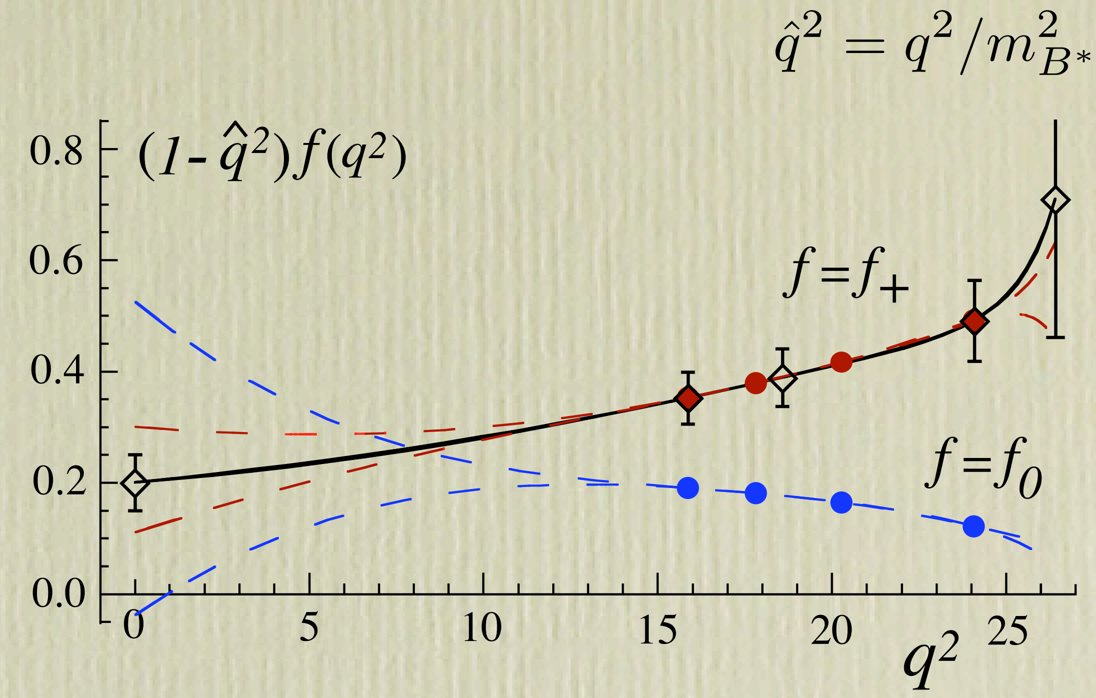
- solve with $\sum_{n=0}^{5} a_n z^n$, for a_0–a_4
- vary a_5 to get bounds $\sum_n a_n^2 \le 1$

$$f_+(t) = F_\pm(t, \{f_0/|V_{ub}|, f_1, f_2, f_3, f_4\})$$



Uncertainties

Bound uncertainty:



- fix $f^i = f^i_{\rm in}, \quad |V_{ub}| = 3.6 \times 10^{-3}$

  ➡ bound uncertainty very small

- compare with 4 lattice points,
  and constraint $f_0(0) = f_+(0)$

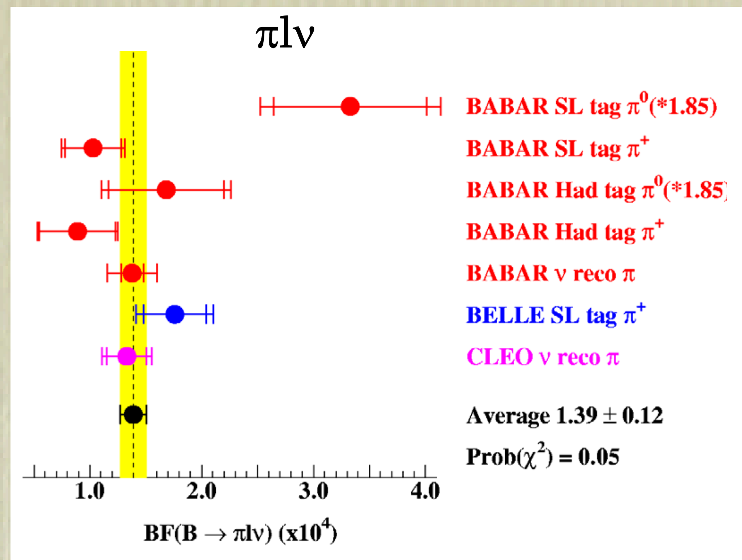
Perturbative uncertainty:

- OPE $\chi^{(0)}$ depends on m_b, order in $\alpha_s(m_b)$, condensates

  ➡ only effects norm., so enters through a_5 , very small

Uncertainty from INPUT POINTS dominates

Method I



- use $\mathrm{Br}(\bar{B}^0 \to \pi^+\ell^-\bar{\nu}) = (1.39 \pm 0.12) \times 10^{-4}$

  J.Dingfelder (CKM'05)

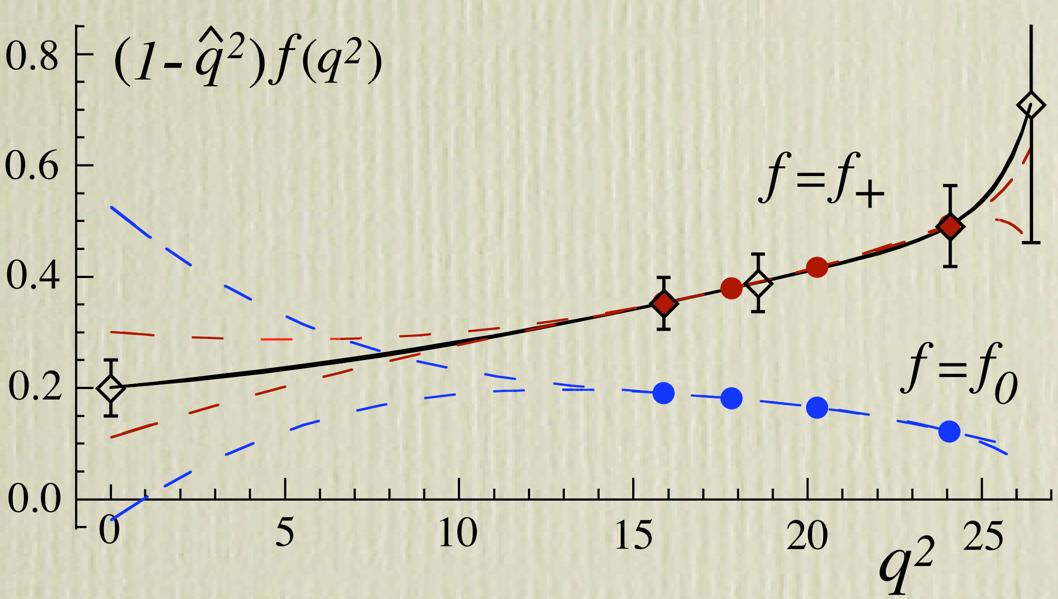
- integrate $\frac{d\Gamma}{dq^2}$ with $f_+(t) = F_\pm(t, \{f_0/|V_{ub}|, f_1, f_2, f_3, f_4\})$
- use Lellouch method to account for theory uncertainty

$$|V_{ub}| = (3.96 \pm \underbrace{0.20}_{\substack{5\% \\ \text{expt}}} \pm \underbrace{0.56}_{\substack{14\% \\ \text{theory}}}) \times 10^{-3}$$

(with $f^i = f^i_{\text{in}}$ $|V_{ub}| = 4.13 \times 10^{-3}$)

Type of Error	Variation From	$\delta\|V_{ub}\|^{\mathrm{Br}}$	$\delta\|V_{ub}\|^{q^2}$
Input Points	1-σ correlated errors	$\pm 14\%$	$\pm 12\%$
Bounds	F_+ versus F_-	$\pm 0.6\%$	$\pm 0.04\%$
m_b^{pole}	4.88 ± 0.40	$\pm 0.1\%$	$\pm 0.2\%$
OPE order	2 loop $\to$ 1 loop	-0.2%	$+0.3\%$

without SCET bound error is $\pm 12\%$



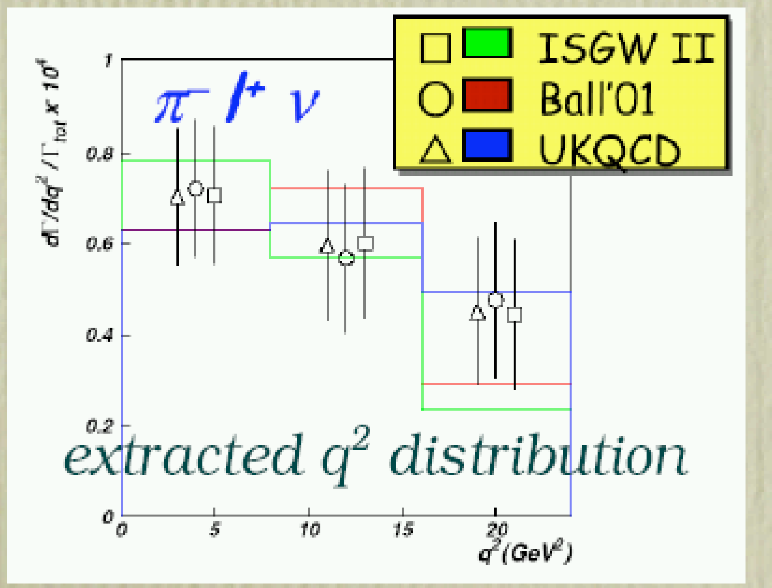
Method II

- use q^2 spectra bins: $(\mathrm{Br}_i^{\exp} \pm \delta\mathrm{Br}_i)$, calculate rate in bins
- use Minuit to minimize χ^2 w.r.t. $|V_{ub}|,\ f^{0-4}$

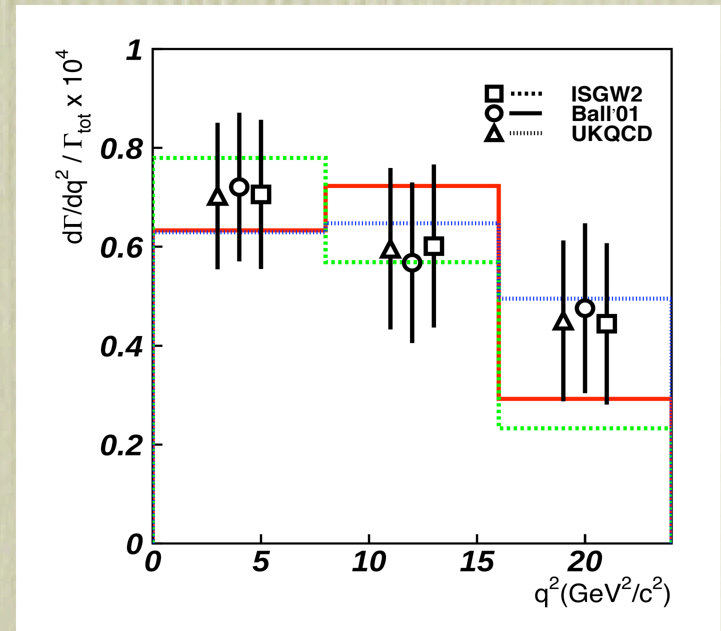
$$\chi^2 = \sum_{i=1}^{17} \frac{[\mathrm{Br}_i^{\exp} - \mathrm{Br}_i(V_{ub}, F_\pm)]^2}{(\delta\mathrm{Br}_i)^2} + \frac{[f_{\rm in}^0 - f^0]^2}{(\delta f^0)^2} + \frac{[f_{\rm in}^4 - f^4]^2}{(\delta f^4)^2} + \sum_{i,j=1}^{3} [f_{\rm in}^i - f^i]\,[f_{\rm in}^j - f^j]\,(E^{-1})_{ij}\,,$$

f^{0-4} input points get adjusted to improve the fit to data (ie. the spectra constrain the theory error)

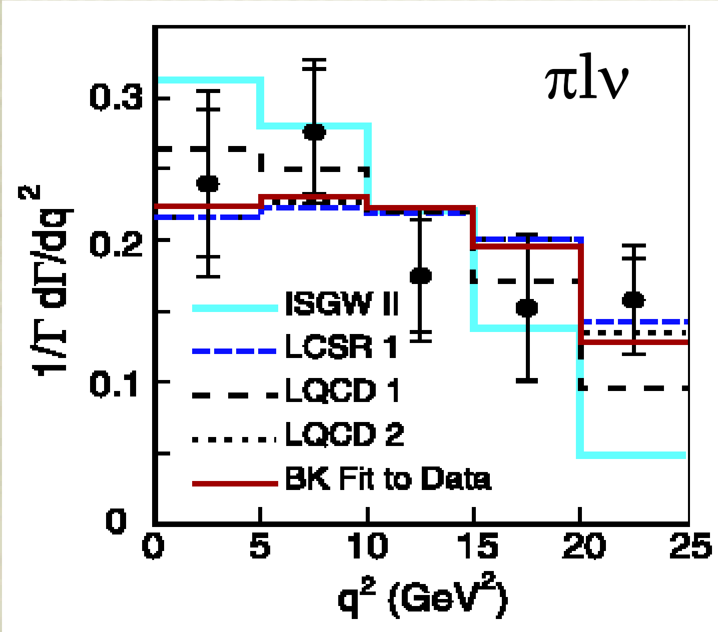
Cleo



Belle



Babar



Fit Results

$$|V_{ub}| = (3.54 \pm 0.47) \times 10^{-3}$$

5% expt   12% theory

$$f_+(0) = 0.227 \pm 0.047$$

Type of Error	Variation From	$\delta\|V_{ub}\|^{\rm Br}$	$\delta\|V_{ub}\|^{q^2}$
Input Points	1-σ correlated errors	$\pm 14\%$	$\pm 12\%$
Bounds	F_+ versus F_-	$\pm 0.6\%$	$\pm 0.04\%$
$m_b^{\rm pole}$	4.88 ± 0.40	$\pm 0.1\%$	$\pm 0.2\%$
OPE order	2 loop $\rightarrow$ 1 loop	-0.2%	$+0.3\%$

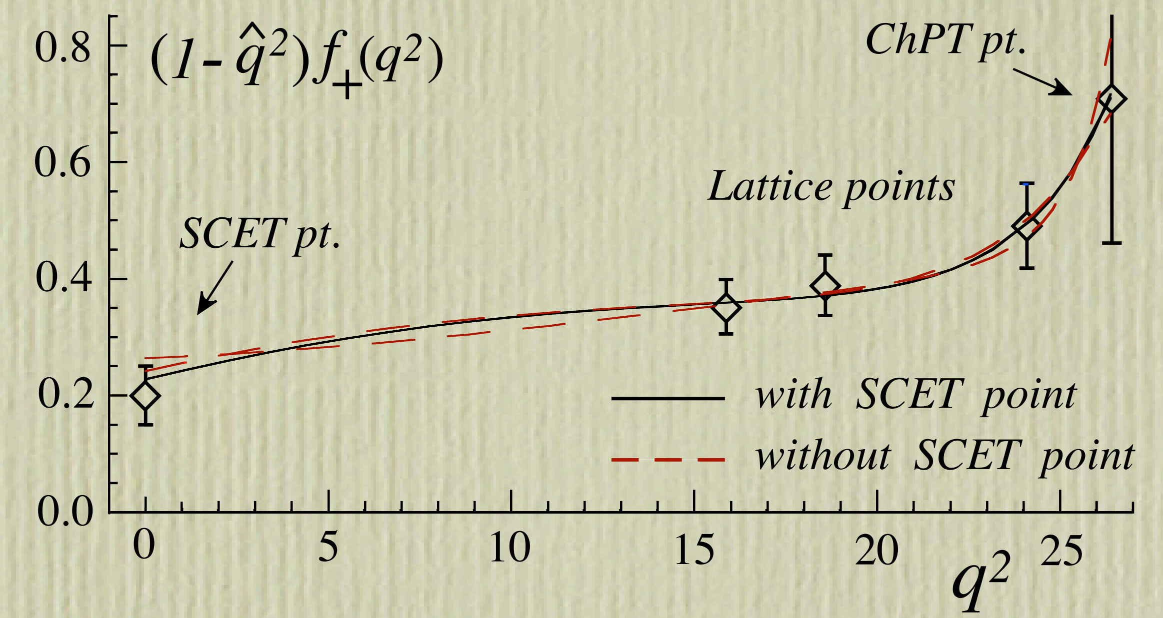
without SCET point:

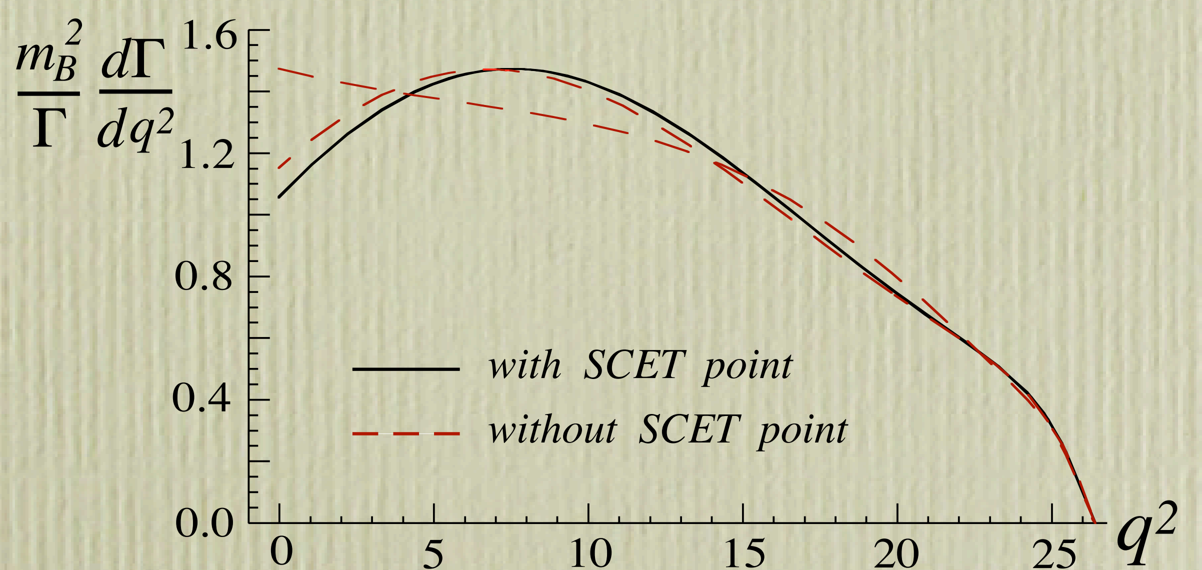
$$|V_{ub}| = (3.56 \pm 0.48) \times 10^{-3}$$

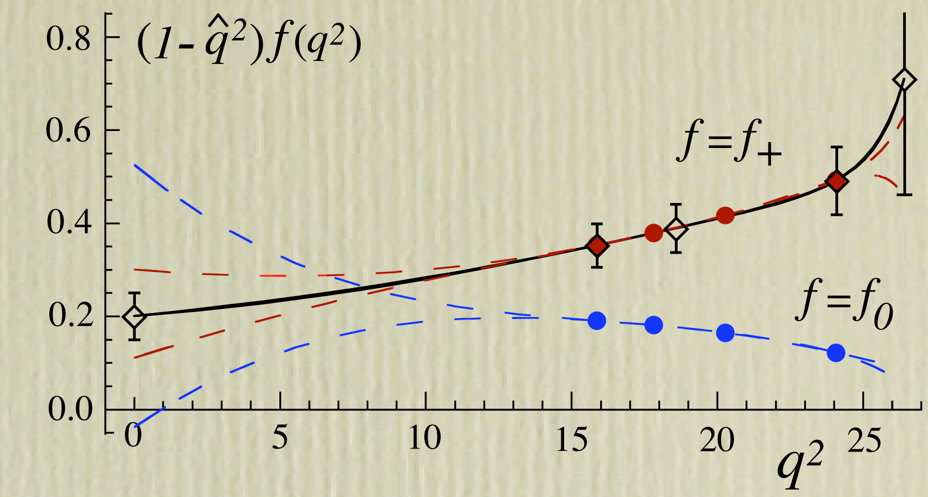
(bound error becomes $\delta|V_{ub}| = \pm 1.8\%$)

$$f_+(0) = 0.25 \pm 0.06$$

Plots for Fit Results







- expt. spectrum prefers a larger form factor in $\sim 5\text{–}10\,\mathrm{GeV}^2$ region
- Here the SCET point constrains the spectrum, but does not change the determination of Vub

Conclude

- Model independent exclusive determination of Vub with 13% total uncertainty can be achieved

$$|V_{ub}| = (3.54 \pm 0.47) \times 10^{-3}$$

- Method II should be redone with with experimental correlation matrices and better lattice correlation matrix

Future (theory errors)

Lattice errors dominate the 12% (and 14%)

An alternative approach is to use symmetry, double ratios, and more experimental input on other decay rates